

Homework 3

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.

Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

Section 1.6

1. Determine whether the following set is a basis of \mathbb{R}^3 .

$$\{(1, -3, -2), (-3, 1, 3), (-2, -10, -2)\}$$

2. Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be distinct vectors of a vector space V . Show that if $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a basis for V then $\{\mathbf{u} + \mathbf{v} + \mathbf{w}, \mathbf{v} + \mathbf{w}, \mathbf{w}\}$ is also a basis for V .
3. Find a basis for

$$W = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : a_2 = a_3 = a_4 \text{ and } a_1 + a_5 = 0\}.$$

What is the dimension of W ?

4. The set of all upper triangular $n \times n$ matrices is a subspace W of $M_{n \times n}(F)$. Find a basis for W . What is the dimension of W .

Section 2.1

5. Show that the *reflection* operation (on \mathbb{R}^2), discussed in class is a linear transformation.