Homework 4

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.
Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

Section 2.1

1. (a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$.
   (b) $T : M_{2\times 3}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ defined by
   $$T\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}.$$
   In each of the above T:
   (i) Show that T is a linear transformation.
   (ii) Find a basis for $N(T)$.
   (iii) Find a basis for $R(T)$.
   (iv) Verify the Dimension Theorem.
   (v) Determine whether T is one-to-one or onto.

2. In the following for $T : \mathbb{R}^2 \to \mathbb{R}^2$, show that T is *not* linear.
   (a) $T(a_1, a_2) = (1, a_2)$
   (b) $T(a_1, a_2) = (a_1, a_1^2)$

3. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear, $T(1, 0) = (1, 4)$, and $T(1, 1) = (2, 5)$. What is $T(2, 3)$? Is T one-to-one?

4. Recall, that $P(\mathbb{R})$ is the set of all polynomials with coefficients in $\mathbb{R}$. Define
   $$T : P(\mathbb{R}) \to P(\mathbb{R}) \text{ by } T(f(x)) = \int_0^x f(t)dt.$$
   Prove that T is linear and one-to-one but not onto.

Section 2.2

5. Define $T : M_{2\times 2}(\mathbb{R}) \to P_2(\mathbb{R})$ by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a + b) + (2d)x + bx^2$. Let $\left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}$ be the basis for $M_{2\times 2}(\mathbb{R})$ and $\{1, x, x^2\}$ be the basis for $P_2(\mathbb{R})$. Compute the matrix of T.