Homework 5

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.
Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

Section 3.3

1. Determine which of the following systems has a solution. Give all solutions if one exists.

   (a)

$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ 2x_1 + x_2 + 3x_3 &= 2 \end{aligned}$$

   (b)

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 + x_2 + 2x_3 &= 3 \\ x_1 - 4x_2 + 7x_3 &= 4 \end{aligned}$$

2. In each system given below:

   - Solve the system $A\mathbf{x} = \mathbf{b}$.
   - Find the null space of A i.e., the solution of the homogeneous system $A\mathbf{x} = 0$.
   - Express the solution set of $A\mathbf{x} = \mathbf{b}$ as a sum of a particular solution of $A\mathbf{x} = \mathbf{b}$ and solutions of $A\mathbf{x} = 0$.

   (a)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & -1 \end{bmatrix},\ b = \begin{bmatrix} 7 \\ -4 \end{bmatrix}.$$

   (b)

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 1 \\ x_2 - x_3 + x_4 &= 1 \end{aligned}$$

Section 4.2

3. Find the determinant of the following matrices. For convenience, in some cases you should use properties of determinants and consider the corresponding row reduced form.

a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 3 & 2 \\ 4 & -8 & 1 \\ 2 & 2 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -2 & 3 & -12 \\ -5 & 12 & -14 & 19 \\ -9 & 22 & -20 & 31 \\ -4 & 9 & -14 & 15 \end{bmatrix}$

Section 4.3

4. Prove that an upper triangular $n \times n$ matrix is invertible if and only if all its diagonal entries are nonzero.

5. A matrix $Q \in M_{n\times n}(\mathbb{R})$ is called orthogonal if $QQ^t = I$ where I is the $n\times n$ identity matrix. Prove that if Q is orthogonal, then $\det(Q) = \pm 1$. (Note that here Q^t is the transpose of the matrix Q.)