Homework 6

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.
Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

1. It can be shown that if A is an $n \times n$ matrix then its characteristic polynomial has the form

$$f(\lambda) = (-1)^n \lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0.$$

(a) Using the above fact, prove that $f(0) = a_0 = \det(A)$.
(b) Hence deduce that $\det(A) =$ product of the eigenvalues of A.
(Hint: If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of A then we also have $f(\lambda) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\cdots(\lambda_n - \lambda)$.)

2. For the given linear operator $T : V \to V$, test for diagonalizability, and if T is diagonalizable, find an ordered basis β for V such that $[T]_\beta$ is diagonal.
a) $V = \mathbb{R}^3$ and $T(a, b, c) = (-4a + 3b - 6c, 6a - 7b + 12c, 6a - 6b + 11c)$

b) $V = P_3(\mathbb{R})$ and $T(f(x)) = xf'(x) + f''(x) - f(2)$

c) $V = P_3(\mathbb{R})$ and $T(f(x)) = f'(x) + f''(x)$.
(Note: To find eigenvalues of T use the standard basis of $\mathbb{R}^3$ or P_3.)

3. For each of the following matrices A, test for diagonalizability. If A is diagonalizable, find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$, then find an expression for A^n.

(a) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{bmatrix}$

4. Using the Cayley-Hamilton Theorem, evaluate $2A^2 - 9A - 10I$ where I is the 2×2 identity matrix and A is the 2×2 matrix given in 3 (a) above.