Homework 8

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.
Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

Section 6.2

1. Let $V = C([0,1])$, the space of continuous functions defined on $[0,1]$. Consider the inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)\ dt$. Let W be the subspace spanned by the linearly independent set $\{t, \sqrt{t}\}$.
   (a) Find an orthonormal basis for W.
   (b) Let $h(t) = t^2$. Use the orthonormal basis obtained in (a) to find the best (closest) approximation of h in W.

Section 6.3

2. For each of the following inner product spaces V and linear operators T on V, find T^* and evaluate $T^*(x)$ for the given vector.
   (a) $V = \mathbb{R}^2$, $T(a,b) = (2a+b, a-3b)$, $x = (3,5)$
   (b)$V = P_1(\mathbb{R})$ with $\langle x, y\rangle = \int_{-1}^1 x(t)y(t)dt$, $T(x) = x'(t)+3x(t)$, $x(t) = 4-2t$.
   (Note: You need to use an orthonormal basis for V in each case. In (b) you can start with the standard basis $\{1, t\}$ of P_1 and orthonormalize it using the Gram-Schmidt process.)

3. Let V be an inner product space, and let $y, z \in V$. Define $T : V \to V$ by $T(x) = \langle x, y\rangle z$ for all $x \in V$. Prove that T is linear. Find an explicit expression for T^*.

4. Consider the data set

$$\{(t_i, y_i)\} = \{(-3,9), (-2,6), (0,2), (1,1)\}.$$

   Use the least squares approximation method to find the best fits with both a linear function and a quadratic function. Compute the error in both cases.