

Homework 9

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.

Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

Section 6.5

1. Prove that a matrix that is both unitary and upper triangular must be a diagonal matrix.
2. Find new coordinates x', y' so that the following quadratic forms can be written as $\lambda_1 x'^2 + \lambda_2 y'^2$. Specify clearly what λ_1 and λ_2 are in each case.
 - (a) $x^2 - 12xy - 4y^2$
 - (b) $x^2 - 2xy + y^2$

3. Let A and B be $n \times n$ matrices that are unitarily equivalent. Prove that

$$\operatorname{tr}(A^*A) = \operatorname{tr}(B^*B).$$

Section 6.7

4. Find a singular value decomposition for each of the following matrices.

(a)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$