Additional Practice Problems for Midterm I

MATH 472

1. Prove that the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$ diverges to ∞.
   (Consider $\lim_{k\to\infty} \frac{k}{k+1}$.)

2. Is the following statement true or false? Explain.
   The series $\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{2^k}\right)$ converges.

3. Test the convergence of the following series
   (a) $\sum_{k=1}^{\infty} \frac{5}{3^k+2}$
   (b) $\sum_{k=1}^{\infty} \frac{1}{2+\sqrt{k}}$

4. Determine whether the following series converges conditionally, converges absolutely or diverges:
   $\sum_k \frac{(-1)^k}{\sqrt{k(k+1)}}$

5. Find the pointwise limit of the sequence $\{f_n\}$ where $f_n : [0, \infty] \to \mathbb{R}$ is given by
   $$f_n(x) = \frac{x^n}{1 + x^{2n}}.$$

6. Suppose that $\{f_n\}$ with $f_n : [2, 5] \to \mathbb{R}$ is defined by $f_n(x) = \frac{x^n}{1+x^{2n}}$. Find $\lim_{n\to\infty} \int_2^5 f_n(x)\, \mathrm{d}x$

7. For each $n \in \mathbb{N}$, consider the functions $f_n : [-1, 1] \to \mathbb{R}$ defined by $f_n(x) = xe^{-nx^2}$.

   (a) Show that $\{f_n\}$ converges uniformly to a differentiable function.

   (b) Show that the limit as $n \to \infty$ and the differentiation process cannot be interchanged.