

# Practice Problems for Midterm I

## MATH 420

- Describe the set determined by the given condition: (a)  $|z - 4i| \geq 4$   
(b)  $|z - 1 + i| = 1$
- Find  
(a)  $\bar{iz}$  (b) the modulus of  $-5 - i\sqrt{11}$
- Write in polar form  
(a)  $i(1 - i\sqrt{3})$  (b)  $2(1 + i\sqrt{3})$
- Use de Moivre's Theorem to show that

$$\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$$

- Find  $(-1 - i)^4$
- Find all values of  
(a)  $(-8i)^{1/3}$  (b)  $(-1)^{1/3}$
- Find  $\lim_{z \rightarrow \infty} \frac{1-z}{z^2+1}$   
(Ans = 0)
- Find  $\lim_{z \rightarrow i} z^2 + 2z$   
(Ans.  $-1 + 2i$ )
- Is the function  $f(z) = \frac{1}{z-i}$  continuous at  $z = i$ ? Why, or why not?  
(Ans. Not continuous at  $z = i$  because  $f(i)$  does not exist)
- Check the C-R equations for  $f(z) = x^3 + i(1 - y)^3$  and find the set where the function is not analytic.  
(Ans. Everywhere except the point  $x = 0, y = 1$  or, everywhere except the number  $i$ .)
- Show that the function  $\log z$  is not continuous along the positive real axis (refer to Lecture 12).
- Using Euler's formula write  $e^{i\pi/2}$  in polar form and find its value.  
(Ans.  $e^{i\pi/2} = \cos \pi/2 + i \sin \pi/2 = i$ )

13. Show that  $e^{2+3\pi i} = -e^2$
14. Find the logarithm of  $-1 - \sqrt{3}i$  and give the principal value.  
(Ans.  $\log(-1 - \sqrt{3}i) = \log 2 + i(4\pi/3 + 2n\pi)$  and  $\text{Log}(-1 - \sqrt{3}i) = \log 2 + i4\pi/3$ )