Series Convergence Tests (JT)

Test	Series $\sum_{n=1}^{\infty} a_n$ Conditions	Converges when	Diverges when	Comments
Divergence			$\lim_{n\to\infty} a_n \neq 0$	Test fails if $\lim_{n\to\infty} a_n = 0$
Integral	let $f(n) = a_n$; $n \geq 1$ and see comments	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	f(x) must be continuous, positive, & decreasing $\forall$ n>#
Comparison	$a_n > 0$ and $b_n > 0$, for all *n*	$\sum_{n=1}^{\infty} b_n$ converges and $b_n \geq a_n$, $\forall$ n>some number	$\sum_{n=1}^{\infty} b_n$ diverges and $b_n \leq a_n$, $\forall$ n>some number	
Limit Comparison	$a_n > 0$ and $b_n > 0$, for all *n*	$\sum_{n=1}^{\infty} b_n$ converges and $\lim_{n\to\infty} \frac{a_n}{b_n} = \# \geq 0$	$\sum_{n=1}^{\infty} b_n$ diverges and $\lim_{n\to\infty} \frac{a_n}{b_n} = \# > 0$ or $= \infty$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ or $\sum_{n=1}^{\infty} (-1)^n b_n$ where $b_n > 0, \forall n$	$\lim_{n\to\infty} b_n = 0$ and $\{b_n\}$ is eventually decreasing	$\lim_{n\to\infty} a_n \neq 0$	
Ratio		$\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = \# < 1$	$\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = \# > 1$ or $= \infty$	Test fails if limit = 1
Root		$\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = \# < 1$	$\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = \# > 1$ or $= \infty$	Test fails if limit = 1