MATH 420

COMPLEX VARIABLES

SESSION no. 1

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Real number system - $\mathbb{R}$

Any real r has a decimal rep.

Eg. $r = 10.1112,\ 0.431,\ 1.27 \ldots$

If $r = \frac{p}{q}$ then r is rational

$2/3,\ 5/1$

Else r is irrational, eg.,

$\sqrt{2}$

Find the roots of $\underbrace{z^2+1}$ ↪ polynomial

i.e. solve for z in

$$z^2+1=0$$
or, $$z^2=-1$$

] No solution in $\mathbb{R}$

Let i be a new kind of no.

$$i = \sqrt{-1}$$ → imaginary unit

Solve $z^2 + 1 = 0$

$$\Rightarrow z^2 = -1$$

$$\Rightarrow z = \pm\sqrt{-1}$$

$$= +i, -i$$

Definition A complex no. is of the form

$$z = a + ib$$

where $a \in \mathbb{R}$, $b \in \mathbb{R}$, $i = \sqrt{-1}$

$\mathrm{Re}(z) = a \rightarrow$ real part

$\mathrm{Im}(z) = b \rightarrow$ imaginary part

Complex number system

$- \mathbb{C}$.

$z = a + ib$, $i = \sqrt{-1}$

If $b = 0$, then
z is purely real, $\mathbb{R} \subseteq \mathbb{C}$

If $a = 0$, then
z is purely imaginary

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Algebraic operations :

① Equality : $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ then

$$z_1 = z_2 \Leftrightarrow x_1 = x_2 \;\&\; y_1 = y_2$$

if and only if

2 Addition : $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ then

$$z_1 + z_2 = x_1 + iy_1 + x_2 + iy_2$$
$$= (x_1 + x_2) + i(y_1 + y_2)$$

$z_1 = -2 + i3 \qquad z_2 = 4 + i3$

$z_1 + z_2 = 2 + i6$

3 Negative of a complex no.

$$z = x + iy \quad , \quad -z = -x - iy$$

$$z_1 = x_1 + iy_1 \quad , \quad z_2 = x_2 + iy_2$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

4 Subtraction

5 Multiplication:

$$i = \sqrt{-1}, \quad i^2 = -1,$$

$$z_1 = x_1 + iy_1 \quad , \quad z_2 = x_2 + iy_2$$

$$\begin{aligned} z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\ &= x_1 x_2 + i x_1 y_2 + i y_1 x_2 - y_1 y_2 \\ &= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2) \end{aligned}$$

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6 Inverse and division:

Let $Z = x + iy$

complex conjugate

$$\frac{1}{Z} = \frac{1}{x+iy} = \frac{x-iy}{\underbrace{(x+iy)}_{a+b}\underbrace{(x-iy)}_{a-b}}$$

$$= \frac{x-iy}{x^2+y^2}$$

$b = iy$

$b^2 = i^2 y^2$

$= -y^2$

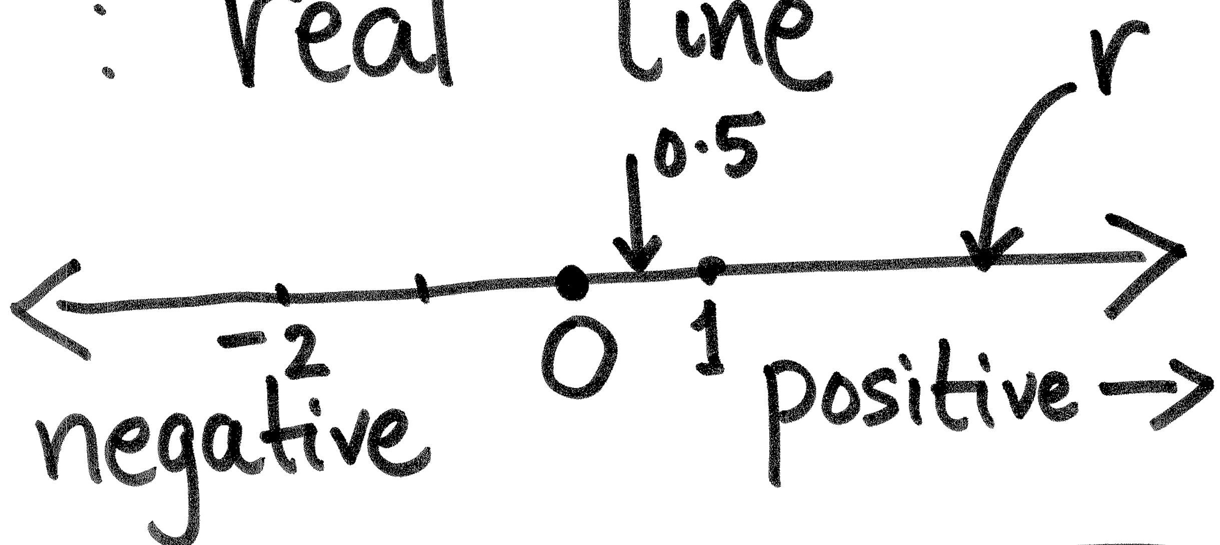
Division : $Z_1 = \cancel{a} x_1 + i y_1$

$Z_2 = x_2 + i y_2$

$$\frac{Z_1}{Z_2} = \frac{(x_1 + i y_1)(x_2 - i y_2)}{x_2^2 + y_2^2}$$

$$= \frac{(x_1 x_2 + y_1 y_2) + i(-x_1 y_2 + y_1 x_2)}{x_2^2 + y_2^2}$$

$\mathbb{R}$: real line



Think of $z = a + ib$ as an ordered pair $(\underbrace{a}_{\mathrm{Re}(z)}, \underbrace{b}_{\mathrm{Im}(z)})$

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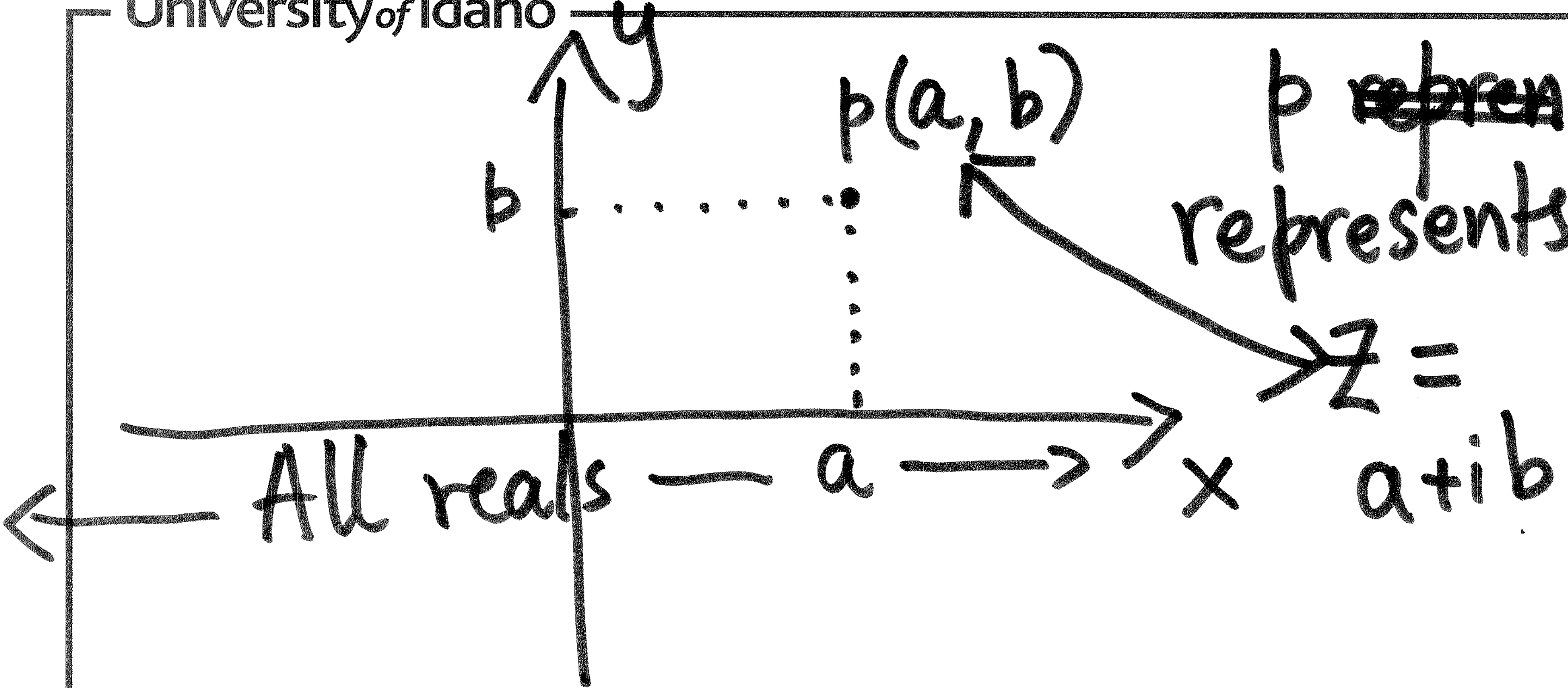

y axis - purely imaginary nos.