MATH 420

COMPLEX VARIABLES

SESSION no. 2

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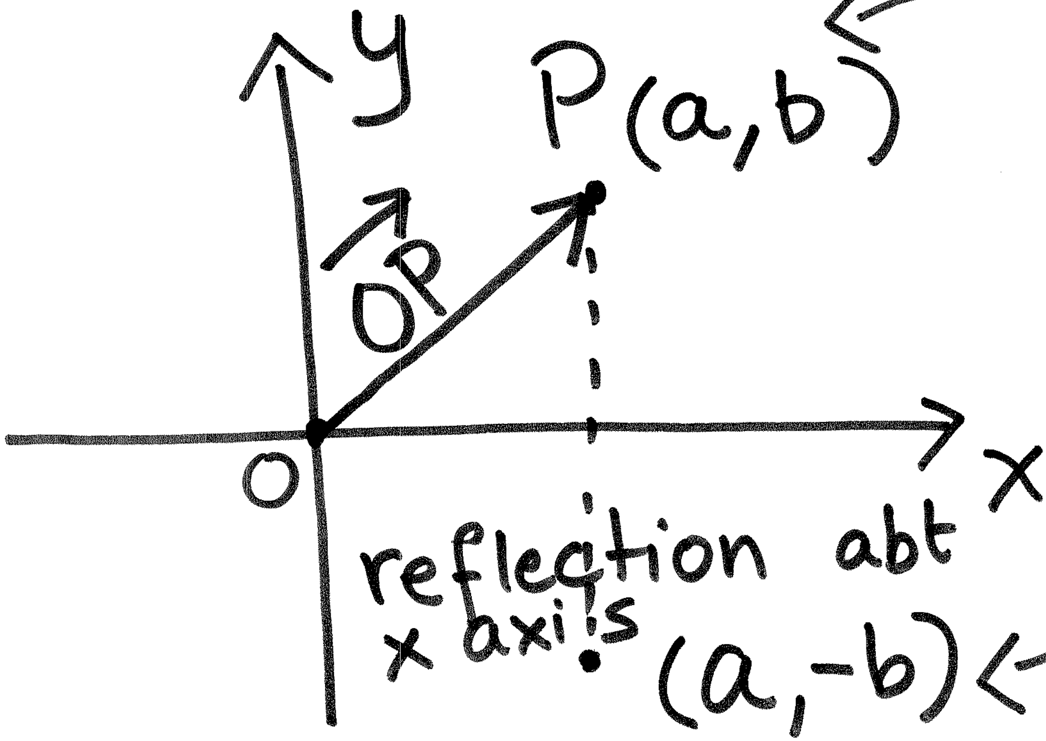
Geometric Representation of complex nos.



$z = a + ib$

$\bar{z} = a - ib$

$|\vec{OP}| = \sqrt{a^2 + b^2}$

x axis – real nos., real axis

y axis – imaginary axis

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CONJUGATE OF A COMPLEX NO

$$z = a + ib$$

Conjugate of $z = a - ib$
$= \bar{z}$

$\bar{z}$ - conjugate of z.

$$z + \bar{z} = a + i\cancel{b} + a - i\cancel{b} = 2a$$

$$\Rightarrow \operatorname{Re} z = a = \boxed{\frac{z + \bar{z}}{2} = \operatorname{Re} z}$$

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$$z - \bar{z} = \cancel{a} + ib - (\cancel{a} - ib)$$
$$= 2ib$$

$$\Rightarrow \quad b = \frac{z - \bar{z}}{2i}$$

$$\Rightarrow \quad \boxed{\operatorname{Im} z = \frac{z - \bar{z}}{2i}}$$

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Properties of conjugate:

$$\overline{z_1 \pm z_2} = \overline{z}_1 \pm \overline{z}_2$$

$$\overline{z_1 z_2} = \overline{z}_1 \overline{z}_2$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}$$

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VECTOR REP. OF A COMPLEX NO.

The directed line segment $\overrightarrow{OP}$ where P is (a, b) is the vector rep. of $z = a+ib$.

Size/Modulus of a complex No.

If $z = a + ib$ then

$$|z| = \sqrt{a^2+b^2}$$

$\hookrightarrow$ modulus of z; mod z

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$$z = a + ib$$

$$z\bar{z} = (a+ib)(a-ib)$$

$$= a^2 - i^2 b^2$$

$$= a^2 + b^2 = |z|^2$$

$$\boxed{|z|^2 = z\bar{z}}$$

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$$\left|\frac{1}{z}\right|^2 = \left(\frac{1}{z}\right)\overline{\left(\frac{1}{z}\right)}$$

$$= \frac{1}{z}\,\frac{1}{\overline{z}}$$

$$= \frac{1}{|z|^2}$$

$$\Rightarrow \left|\frac{1}{z}\right| = \frac{1}{|z|}$$

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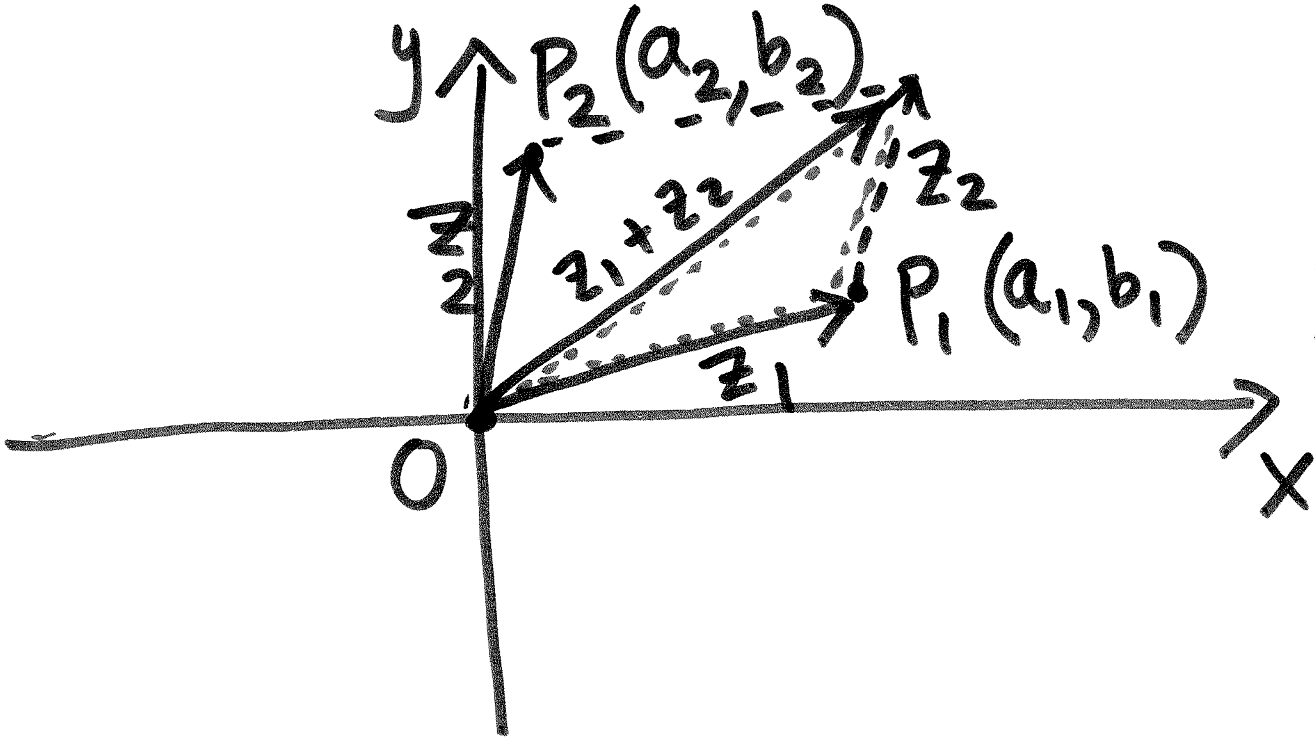
TRIANGLE INEQUALITY

If z_1 & z_2 are two nos. :

$$|z_1 + z_2| \leq |z_1| + |z_2|$$



$z_1 = a_1 + ib_1$

$z_2 = a_2 + ib_2$

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Example: Reduce the following

$$\frac{4+i}{2-3i}$$

$$\frac{4+i}{2-3i}\;\frac{2+3i}{2+3i} = \frac{8+12i+2i+i^{2}3}{4+9}$$

$$= \frac{5+i14}{13}$$

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Example: $z_1 = -1 + i$, $z_2 = 3 + 2i$

Find $|z_1 - z_2|$

$$z_1 - z_2 = (-1-3) + i(1-2)$$

$$= -4 - i$$

$$|z_1 - z_2| = \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1}$$

$$= \sqrt{17}$$

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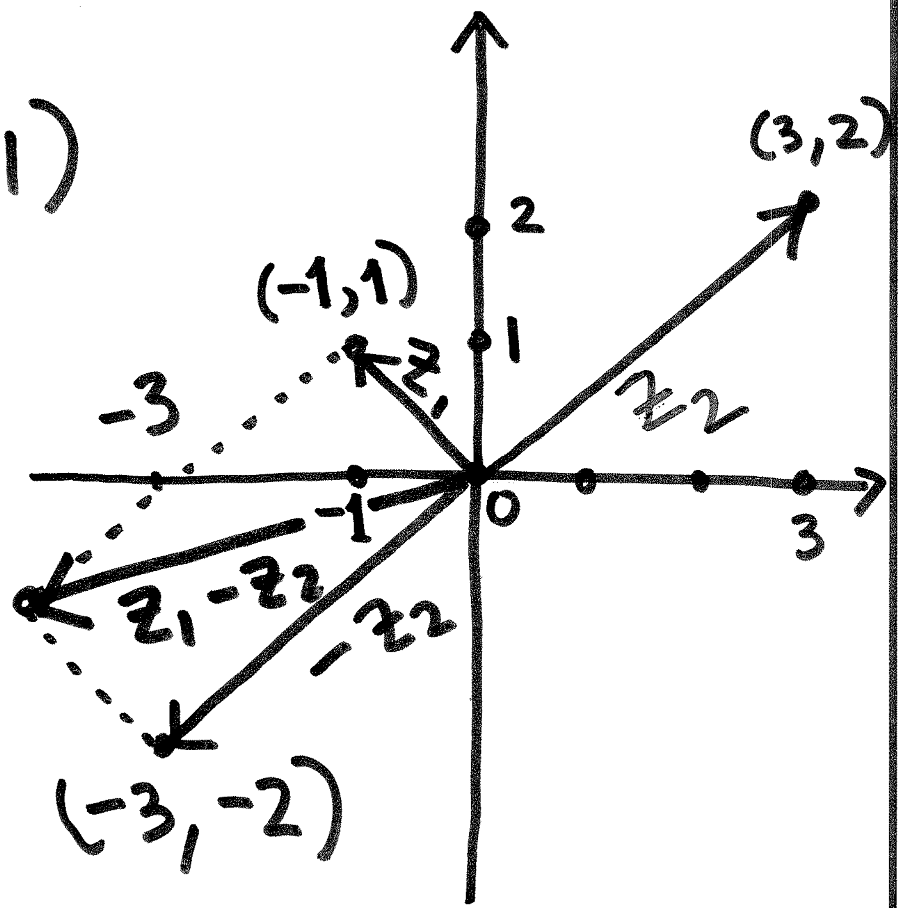
$$z_1 - z_2 \equiv (-4,-1)$$



<u>Example</u> : Show that z satisfying

$$|z| < 2$$

lies within a circle of radius 2 centered at $(0,0)$.

$$z = x+iy \; ; \quad |z| = 2$$

$$\Rightarrow |z|^2 = 4$$

$$\Rightarrow x^2 + y^2 = 4$$

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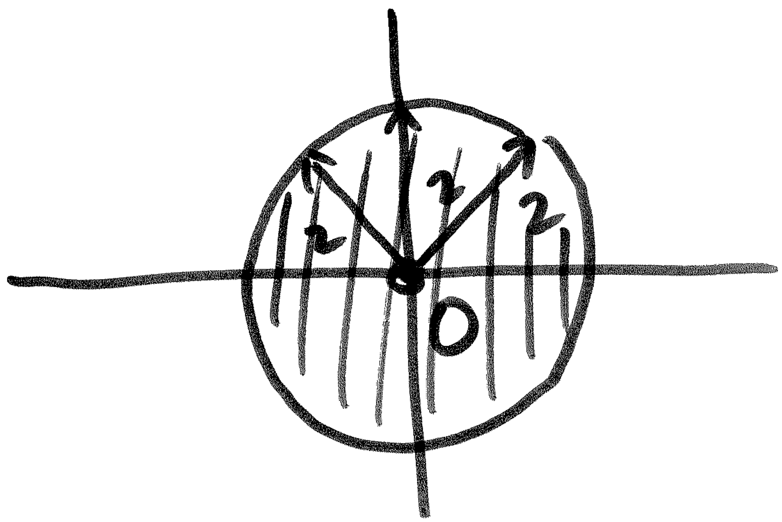
Therefore, $|z| < 2$

$\Rightarrow x^2 + y^2 < 4$

$\Rightarrow$ z lies within the circle centered at (0,0) and of radius 2



<u>Example</u>: Show that if $|z|<2$ then $|z^3+3z^2-2z+1| < 25$

$$|z^3+3z^2-2z+1| \overset{\text{by TRIANGLE}}{\leq} |z^3|+|3z^2|+|-2z|+|1|$$

$$= |z|^3+3|z|^2+2|z|+1$$

$$< 2^3+3(2^2)+2(2)+1 = 25$$

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Example Show that if z satisfies $|z-1+i|=1$ then z lies on a circle.

Let $z = x+iy$. Then

$$|z-1+i| = 1$$

$$\Rightarrow |x+iy-1+i| = 1$$

$$\Rightarrow |(x-1)+i(y+1)|^2 = 1$$

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$$(x-1)^2 + (y+1)^2 = 1$$

This is a circle centered at $(1,-1)$ and with radius $= 1$