MATH 420

COMPLEX VARIABLES

SESSION no. 7

1

Limits of Functions

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$$\lim_{z \to z_0} f(z) = w_0$$

(the limit of f as z approaches z_0 is w_0):

$f(z)$ can be made arbitrarily close to w_0 provided z is close enough to z_0

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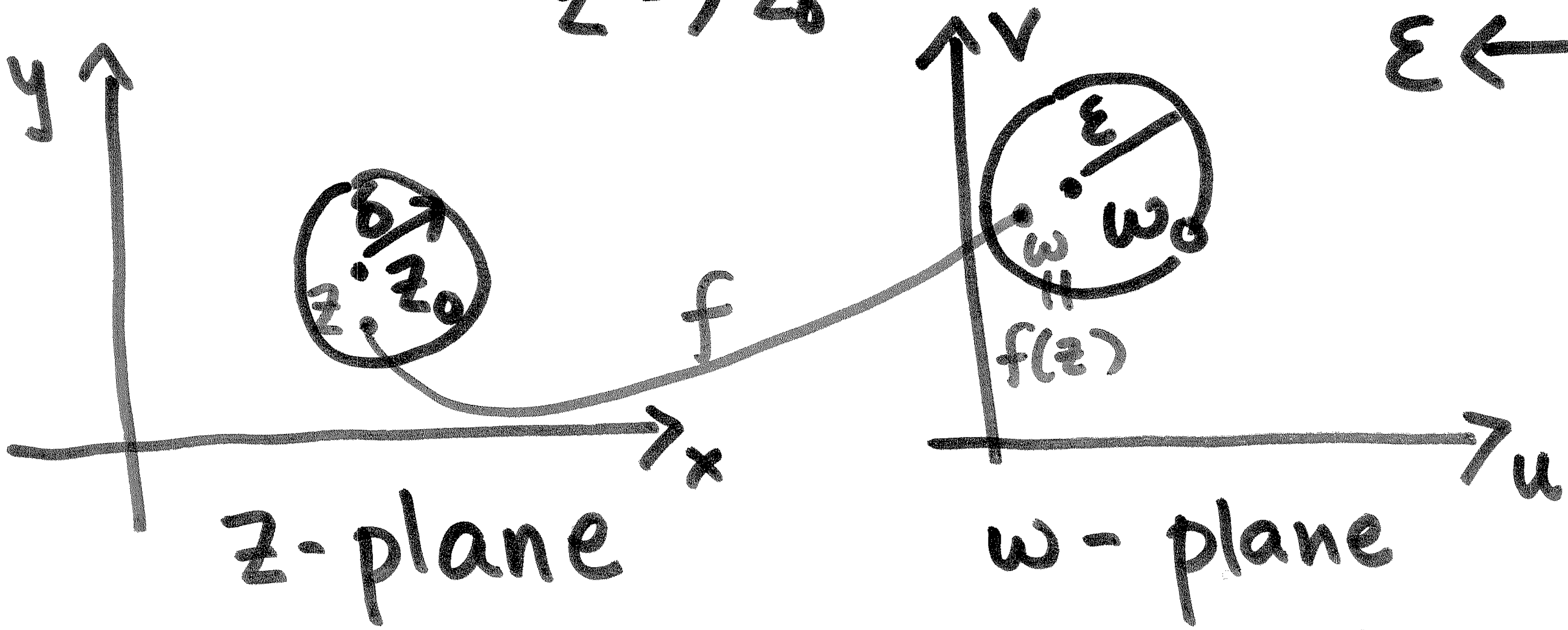
$$\lim_{z \to z_0} f(z) = w_0$$

$\varepsilon \longleftrightarrow \delta$



Given ε, there exists δ s.t.

$$|f(z) - w_0| < \varepsilon \text{ whenever } |z - z_0| < \delta$$

The limit is independent of the path in which z approaches z_0.

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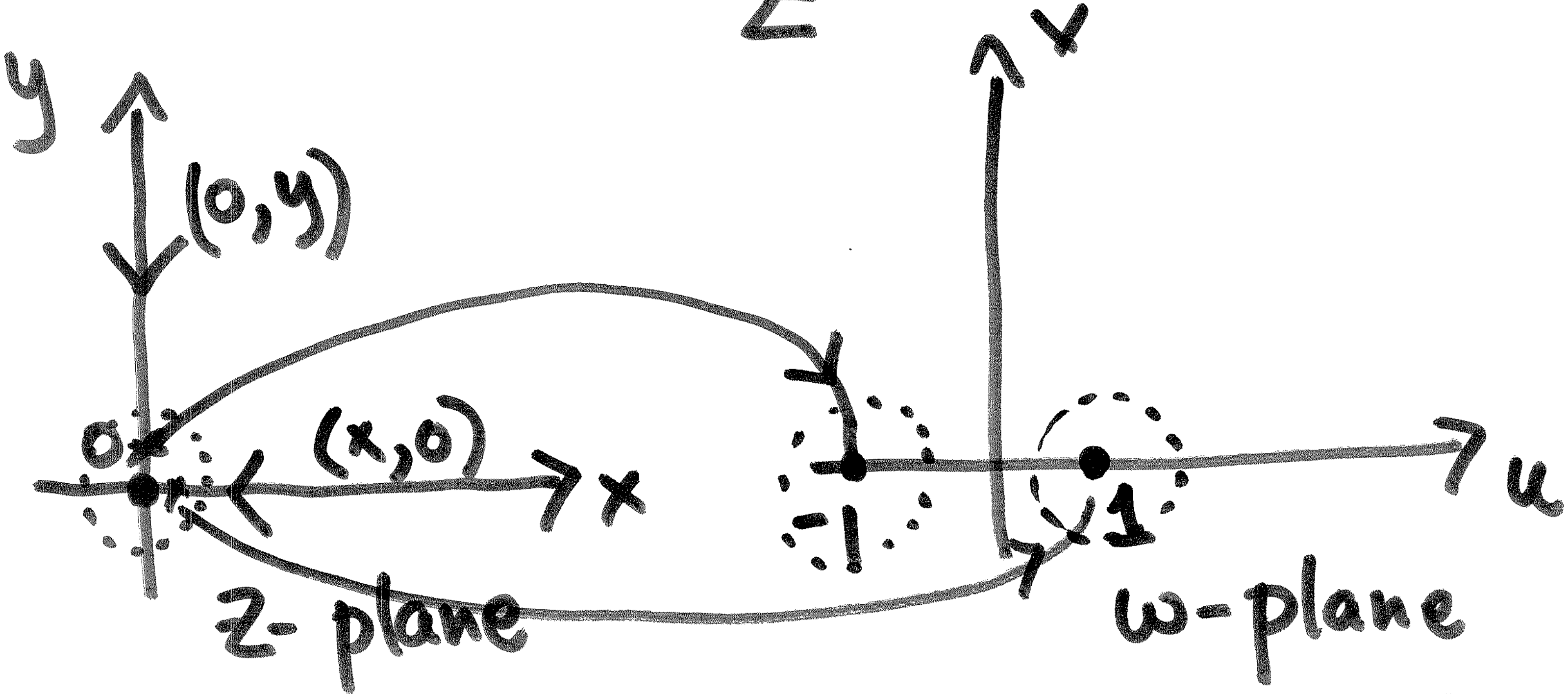
Ex. $f(z) = \dfrac{\bar{z}}{z}$ as $z \to 0$



1) Approach 0 along the real axis

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$$f(z) = \frac{\bar{z}}{z} = \frac{x-iy}{x+iy} = \frac{x}{x} = 1$$

2) Approach 0 along the imaginary axis

$$f(z) = \frac{x-iy}{x+iy} = \frac{-iy}{iy} = -1$$

$f(z)$ does not have a limit at $z \to 0$

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Theorems on Limits

Suppose

$$\lim_{z \to z_0} f_1(z) = w_1 \text{ and } \lim_{z \to z_0} f_2(z) = w_2$$

Then

$$\lim_{z \to z_0} (f_1 \pm f_2) = w_1 \pm w_2$$

$$\lim_{z \to z_0} (f_1 f_2) = w_1 w_2$$

$$\lim_{z \to z_0} f_1/f_2 = \frac{w_1}{w_2} \text{ provided } w_2 \neq 0$$

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Evaluate $\lim_{z \to 1+i} z^2 - 5z + 10$

$= \lim_{z \to 1+i} z^2 + \lim_{z \to 1+i} (-5z) + \lim_{z \to 1+i} 10$

$= (1+i)^2 - 5(1+i) + 10$

$= 1 + 2i - 1 - 5 - 5i + 10$

$= 5 - 3i$

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Limits involving infinity

$$\lim_{z\to z_0} f(z) = \infty \iff \lim_{z\to z_0} \frac{1}{f(z)} = 0$$

$$\lim_{z\to \infty} f(z) = w_0 \iff \lim_{z\to 0} f\left(\frac{1}{z}\right) = w_0$$

$$\lim_{z\to \infty} f(z) = \infty \iff \lim_{z\to 0} \frac{1}{f\left(\frac{1}{z}\right)} = 0$$

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Ex. $\lim_{z \to \infty} \frac{2z+i}{z+1} = 2$

$$\lim_{z \to 0} \frac{2/z + i}{1/z + 1} = \lim_{z \to 0} \frac{2+iz}{1+z}$$

$$= 2$$

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Ex. $\lim\limits_{z\to\infty} \dfrac{2z^3-1}{z^2+1} = \infty$ since

$$\lim_{z\to 0} \frac{\frac{1}{z^2}+1}{\frac{2}{z^3}-1} \cdot \frac{z^3}{z^3} = \lim_{z\to 0} \frac{z+z^3}{2-z^3}$$

$$= 0$$