MATH 420

COMPLEX VARIABLES

SESSION no. 10

University of Idaho HW3

3. $f(z) = (2x^2+y) + i(y^2-x)$

Find the points where f is not analytic.

5. $u(x,y) = 2x(1-y)$ Find $f = u + iv$ s.t. f is analytic.

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1. $\frac{\partial u}{\partial x} = 2(1-y)$ $\frac{\partial u}{\partial y} = ?$

2. $\frac{\partial u}{\partial x} = 2(1-y) = \frac{\partial v}{\partial y}$

3. $\int \frac{\partial v}{\partial y} dy = v = \int 2(1-y)dy + F(x)$

4. Find $\frac{\partial v}{\partial x}$ using 3. Set it equal to $-\frac{\partial u}{\partial y}$. Find F.

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Elementary functions

1. $P(z) = a_0 + a_1 z + \cdots + a_n z^n$

$a_0, \ldots, a_n$ are complex constants.
n - degree of the polynomial

2. exp, sin, cos ...

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

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$i, i^2 = -1, i^3 = -i, i^4 = 1, i^5 = i$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$i\sin\theta = i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!} - \cdots$$

$$\boxed{i\sin\theta = i\theta + \frac{i^3\theta^3}{3!} + \frac{i^5\theta^5}{5!} - \cdots}$$

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$$\cos\theta = 1 + \frac{i^2\theta^2}{2!} + \frac{i^4\theta^4}{4!} + \cdots$$

$$\cos\theta + i\sin\theta = 1 + i\theta + \frac{i^2\theta^2}{2!} + \frac{i^3\theta^3}{3!} + \frac{i^4\theta^4}{4!} + \cdots$$

$$= e^{i\theta}$$

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Euler's Formula:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$z = r\left(\cos\theta + i\sin\theta\right)^{+2n\pi}_{+2n\pi} \quad \text{polar}$$

$$= re^{i\theta} = re^{i(\theta+2n\pi)}$$

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$z = x + iy$

$$w = e^z = e^{x+iy} = e^x e^{iy}$$

$$|e^z| = e^x$$

$$\arg(e^z) = y$$

$$= \left(1 + x + \frac{x^2}{2!} + \cdots\right)\left(1 + iy + \frac{(iy)^2}{2!} + \cdots\right)$$

$$= 1 + (x+iy) + \frac{(x+iy)^2}{2!} + \cdots$$

$$= 1 + z + \frac{z^2}{2!} + \cdots$$