

MATH 420

COMPLEX VARIABLES

SESSION no. 23

$$f(z) = \frac{-1}{(z-1)(z-2)}$$

Not analytic at $z=1, 2$

Expand f about $z=1$:

$$\begin{aligned} f(z) &= \frac{1}{z-1} \frac{1}{2-z} = \frac{1}{z-1} \frac{1}{1-(z-1)} \\ &= \frac{1}{z-1} [1 + (z-1) + (z-1)^2 + \dots]; \quad |z-1| < 1 \end{aligned}$$

$$= -\frac{1}{z-2} + 1 - (z-2) + (z-2)^2 - \dots$$

Ex: Expand $f(z) = (z-3) \sin\left(\frac{1}{z+2}\right)$

about $z = -2$.

$$f(z) = (z+2-5) \sin\left(\frac{1}{z+2}\right)$$

$$= \underbrace{(z+2-5)}_{\dots} \left[\frac{1}{z+2} - \frac{1}{3!(z+2)^3} + \frac{1}{5!(z+2)^5} - \dots \right]$$

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$$= 1 - \frac{5}{z+2} - \frac{1}{3!(z+2)^2} + \frac{5}{3!(z+2)^3} \dots$$

powers of $(z - (-2))$

Valid for all $z \neq -2$.

$$= \frac{1}{z-1} + \underbrace{1 + (z-1) + (z-1)^2 + \dots}_{\text{non-negative powers of } z-1}$$

$(z-1)^{-1}$ → negative power

$f = \frac{-1}{(z-1)(z-2)}$; expand about 2

$$= \frac{1}{z-2} \frac{-1}{(z-1)} = \frac{1}{z-2} \frac{-1}{1+(z-2)}$$

$$= -\frac{1}{z-2} [1 - (z-2) + (z-2)^2 - (z-2)^3 + \dots];$$

$|z-2| < 1$

Ex

$f(z) = \frac{1}{z-2}$; Expand about $z=0$

f is analytic at 0 .

f is analytic for z :

a) $|z| < 2$
 $D_1 \rightarrow$

b) $|z| > 2$ D_2



$$f = \frac{1}{z-2} = \frac{-1}{2-z} = \frac{-1}{2(1-\frac{z}{2})}$$
$$= -\frac{1}{2} \left(1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots \right)$$

valid for $\frac{|z|}{2} < 1$

$$\Rightarrow |z| < 2$$

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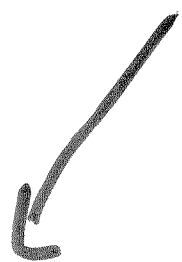
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About $z=0$, outside $|z|=2$ $|z| > 2$ $f(z)$

$$= \frac{1}{z-2}$$

$$= \frac{1}{z(1-\frac{2}{z})}$$

$$= \frac{1}{z} \frac{1}{1-\frac{2}{z}} = \frac{1}{z} \left[1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} \dots \right]$$



valid

$$\frac{2}{|z|} \neq 1 \Rightarrow |z| > 2$$

$$= \frac{1}{z} + \frac{2}{z^2} + \frac{4}{z^3} \dots$$

only negative powers of z


Singularities

Defn: A point z_0 at which $f(z)$ fails to be analytic is called a singularity or a singular point of f .

Isolated singularity: z_0 is an isolated singularity if the disc $|z - z_0| < R$ has no singularities for some R .

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$$f(z) = \frac{1}{z-2} ; z=2 \text{ is a singularity isolated}$$

does not  any R
contain any singularities

$$f(z) = \frac{1}{z^2+4} = \frac{1}{(z-2i)(z+2i)}$$

$2i, -2i$ are isolated singularities

