MATH 420

COMPLEX VARIABLES

SESSION no. 34

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$z = x + iy$

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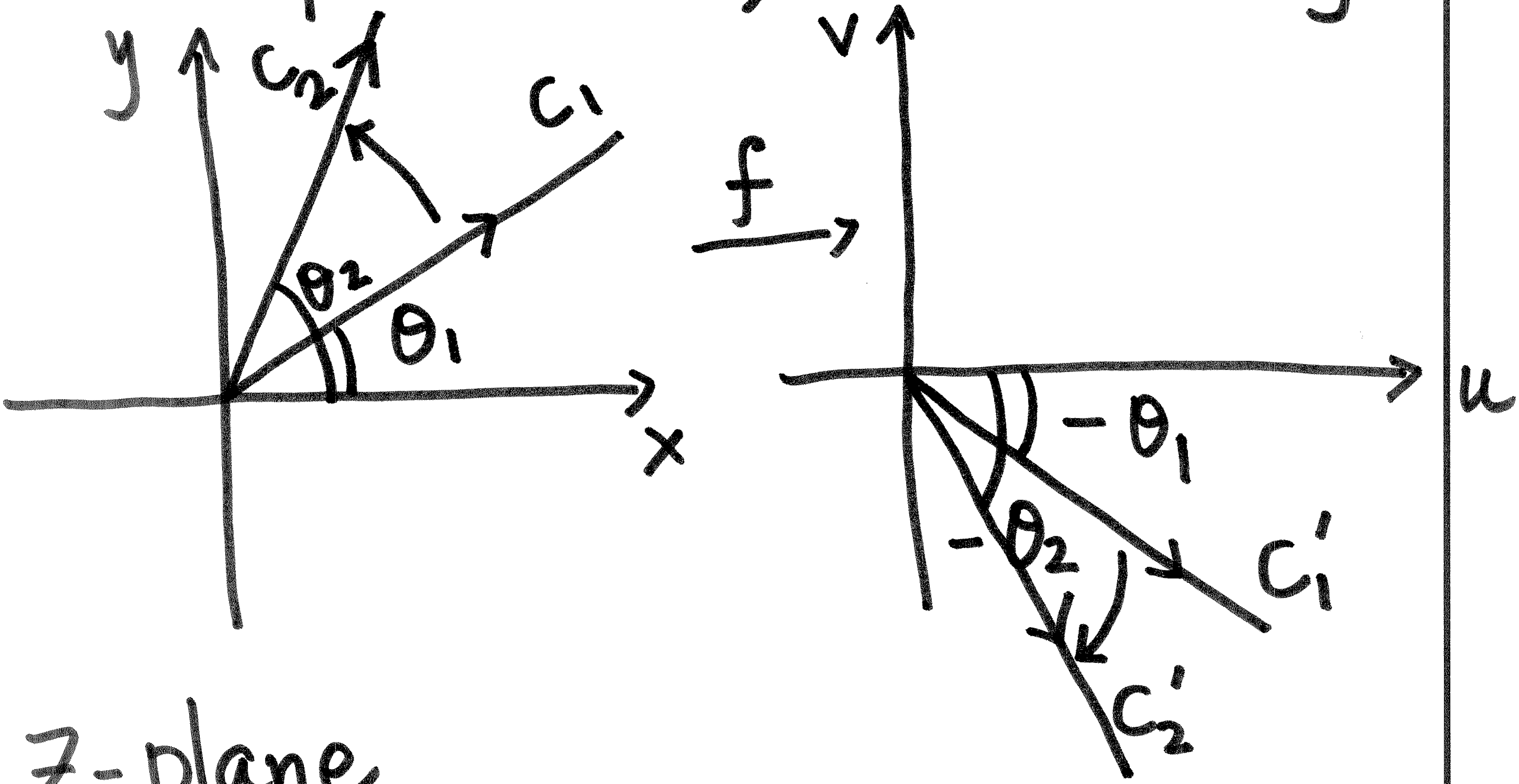
Example: $f(z) = \bar{z} = x - iy$



z-plane

w-plane

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Angle between C_1 & C_2 =
$\theta_2 - \theta_1$

Angle between C_1' & C_2' =
$-\theta_2 - (-\theta_1) = -(\theta_2 - \theta_1)$

$f = \bar{z}$ is isogonal;
not conformal

Check that f is NOT analytic.

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If f is analytic and $f'(z_0) \neq 0$ then f is conformal at z_0.

If $f'(z_0) = 0$ then z_0 is said to be a critical point

Example : $f(z) = z^2$

$f'(0) = 0 \Rightarrow 0$ is a critical point; f is analytic everywhere

f is conformal everywhere except at the origin.

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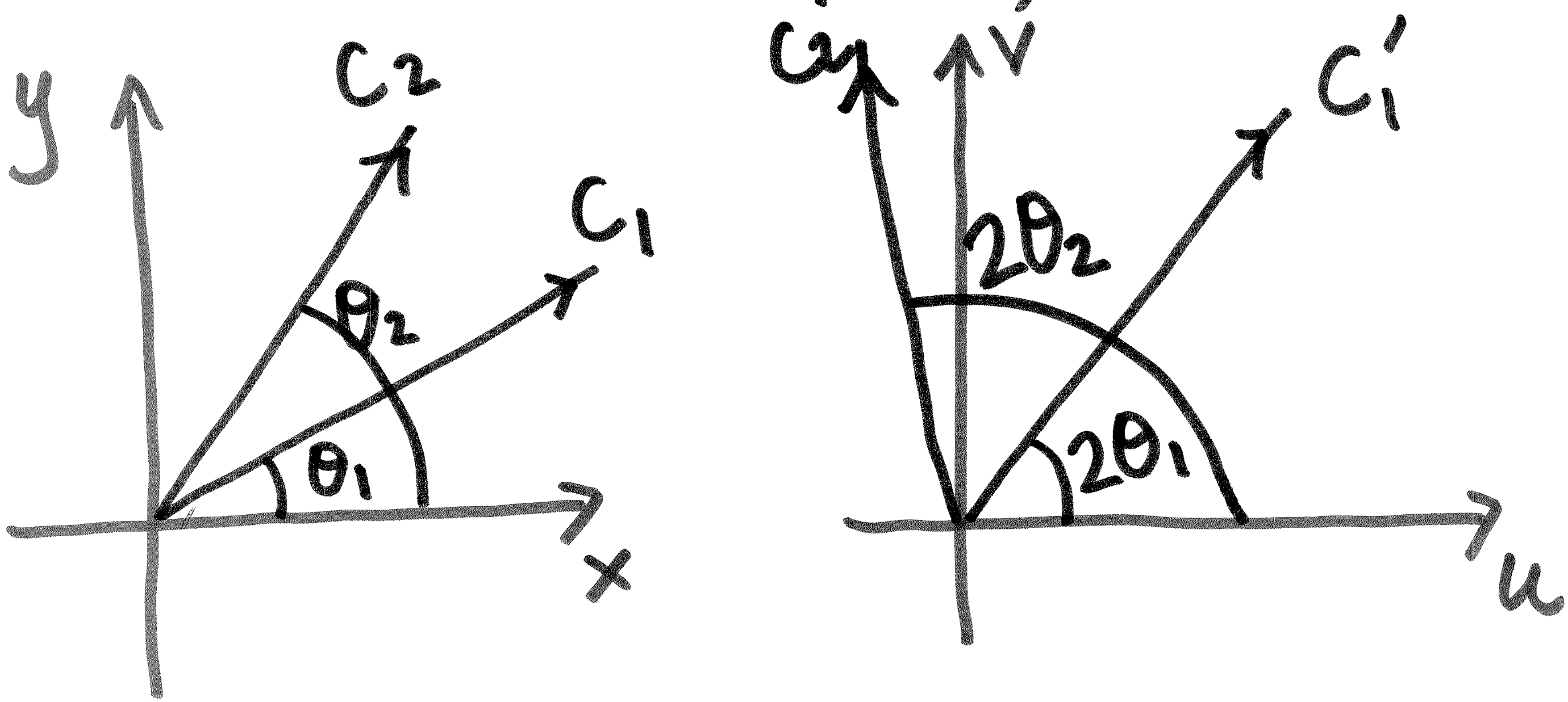
$$z = re^{i\theta}\,;\ f(z) = z^2$$



C_1: $z = re^{i\theta_1}\,;\ r>0 \xrightarrow{f} C_1' : r^2 e^{i2\theta_1}$

C_2: $re^{i\theta_2}\,;\ r>0 \longrightarrow C_2' : r^2 e^{i2\theta_2}$

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For $f = z^2$ the angle between 2 lines passing through $z=0$ have the~~ir~~ angle~~s~~ between them double~~d~~.

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Applications of Conformal Mapping.

H: a real valued function in x and y. If H satisfies

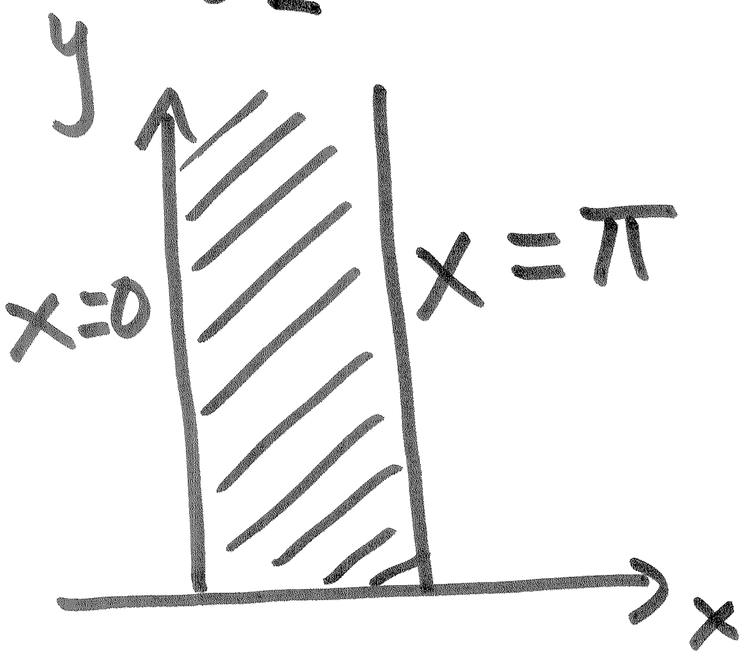
$$H_{xx} + H_{yy} = 0$$

then H is harmonic.

The equation is Laplace's Equation.

Example: $T(x,y) = e^{-y}\sin x$ is the temp. of a plate:

$$0 \le x \le \pi, \quad y > 0$$



$$T(0,y) = 0$$

$$T(\pi,y) = 0$$

9 $T(x,y) = e^{-y}\sin x$

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$T_x = e^{-y}\cos x$, $T_{xx} = -e^{-y}\sin x$

$T_y = -e^{-y}\sin x$, $T_{yy} = e^{-y}\sin x$

$T_{xx} + T_{yy} = 0 \Rightarrow T$ satisfies Laplace's equation.

Properties of harmonic funcs.

Thm: If $f = u + iv$ is an analytic function then u and v are both harmonic.

[v is said to be the harmonic conjugate of u]

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University of Idaho $f = u + iv$ – analytic

<u>Proof</u>: By C-R eqs.

$$u_x = v_y \; ; \; u_y = -v_x$$

Differentiate w.r.t. x:

$$u_{xx} = v_{xy} \; ; \; u_{xy} = -v_{xx}$$

Differentiate w.r.t. y:

$$u_{yx} = v_{yy} \; ; \; u_{yy} = -v_{yx}$$

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(i) $u_{xx} + u_{yy} = v_{xy} - v_{yx} = 0$

(ii) $v_{xx} + v_{yy} = 0$

$\Rightarrow$ u & v are harmonic