MATH 471

INTRODUCTION TO ANALYSIS I

SESSION no. 1

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Preliminary topics

Sets:

subsets, union, intersection, complement

Functions:

domain, image/range

onto function (surjection)

one-to-one (injection)

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Bijection : one-one & onto

inverse function

composition of functions

$f(x) = 1 + \sin x \quad f(0) = 1$

$g(x) = x^2$

$g \circ f(x) = g(f(x)) = g(1 + \sin x)$

$= (1 + \sin x)^2$

composition

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$\mathbb{R}$: the set of all real numbers

[Foundation of analysis]

$\mathbb{N}$: the set of natural nos.

$\{1, 2, 3, \ldots\}$

$\mathbb{Z}$: the set of all integers

$\{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$

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A : a set

$x \in A$

$\in$: belongs to

$\notin$: does not belong to

$\exists$: there exists

s.t. : such that

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Upper & lower bounds

Def: A set $A \subseteq \mathbb{R}$ is **bounded above** if $\exists$ $N \in \mathbb{R}$ s.t. $x \leq N$, $x \in A$.

Ex: $A = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$

$N = 1, 2, 1000, 7, \dots, 200.4$

A is bounded above

The no. N is called an upper bound for A.

Def: A set $A \subseteq \mathbb{R}$ is <u>bounded</u> <u>below</u> if $\exists M \in \mathbb{R}$ s.t. $x \geqslant M$, $x \in A$.

$M = 0$ in the example.

M is called a lower bound for A.

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Least upper bound

Def Given a set $A \subseteq \mathbb{R}$, a number $c \in \mathbb{R}$ is the least upper bound (l.u.b.) if

a) c is an upper bound for A

b) any number smaller than c is no longer an upper bound for A

g.l.b.

$A = \{1, \frac{1}{2}, \frac{1}{3}, \ldots\}$

$C = 1$ is the l.u.b.

Def: A no. ℓ is the greatest lower bound for A if

g.l.b.

a) ℓ is a lower bound for A

b) any no. larger than ℓ is no longer a lower bound

$\ell = 0$ is the g.l.b.

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Remark

The l.u.b. and the g.l.b. need not belong to the set.

$$A = \{1, \tfrac{1}{2}, \tfrac{1}{3}, \dots\}$$

$$\text{l.u.b.} = 1 \in A$$

$$\text{g.l.b.} = 0 \notin A$$

$A = \{1, 2, 3, \ldots\}$

A is not bounded above,

$\nexists$ a l.u.b.

A is bounded below

$M = -1, 1, -100, -20.5$

g.l.b. $= 1$.