MATH 471

INTRODUCTION TO ANALYSIS I

SESSION no. 2

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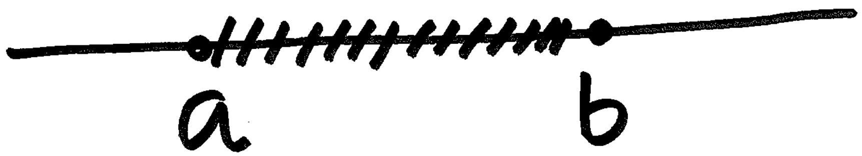
l.u.b & g.l.b.

$$(a, b) = \{x \in \mathbb{R} \text{ s.t. } a < x < b\}$$



l.u.b $= b$

g.l.b $= a$

$$[a, b) = \{x \in \mathbb{R} \text{ s.t. } a \leq x < b\}$$

l.u.b $= b$; g.l.b. $= a$

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$:$ such that or s.t.

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$$(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$$

$$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

l.u.b is also called supremum sup

g.l.b is also called infimum inf

l.u.b & g.l.b of functions

$$f : \underset{\subseteq \mathbb{R}}{A} \longrightarrow \underset{\subseteq \mathbb{R}}{B}$$

$$\text{range}(f) = f(A) = \{ b \in B : f(a) = b \text{ for some } a \in A \}$$

Think of $f(A)$ as a set

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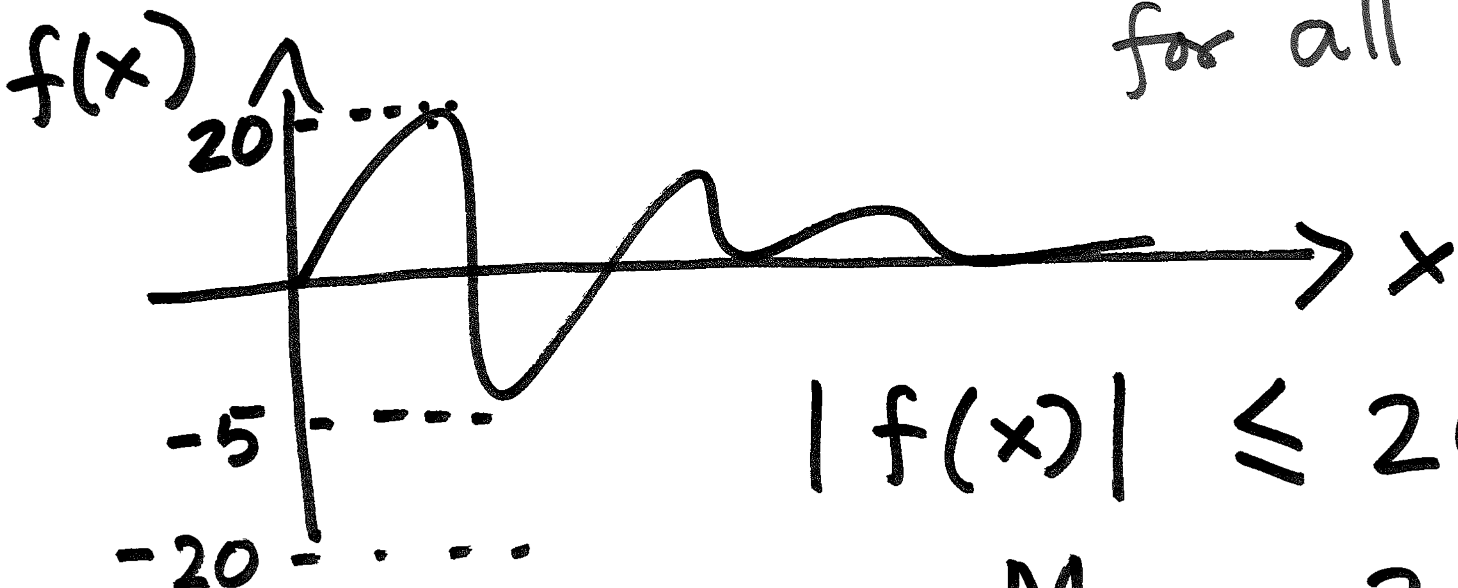
Def f is <u>bounded</u> if $\exists M_1 \in \mathbb{R}$ s.t. $|f(x)| \leq M_1 \quad \forall x \in A$.

for all



$$|f(x)| \leq 20$$

$$M_1 = 20$$

also $\quad M_1 = 25$

$f: A \longrightarrow B$

Def f is <u>bounded above</u> if $\exists\, M_2 \in \mathbb{R}$ s.t. $f(x) \leq M_2$ $\forall\, x \in A$.

Def f is <u>bounded below</u> if $\exists\, M_3 \in \mathbb{R}$ s.t. $f(x) \geq M_3$ $\forall\, x \in A$.

$f: A \to B$

$\inf f := \text{g.l.b } f(A)$

$\sup f := \text{l.u.b } f(A)$

<u>Example1</u> ; $f(x) = \sin x$

$f: \mathbb{R} \longrightarrow [-1, 1]$

$|f(x)| \leq 1$; f is bounded

$M_2 = 1, 5, 1000$ etc. ; f is bounded above

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$M_3 = -1, -20, \ldots$

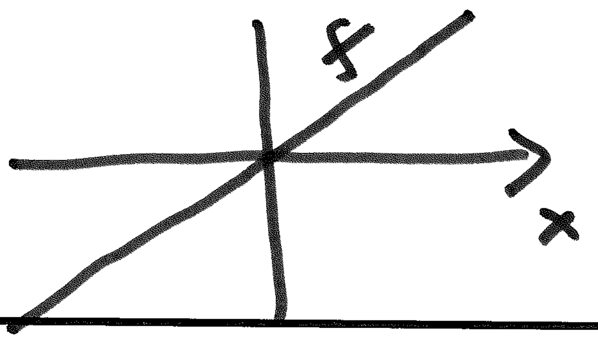
$\cancel{|f(x)| \le M_3}$; f is bounded below

$f(x) \geq M_3$

$\inf f \quad f = -1 \qquad \sup f = 1$

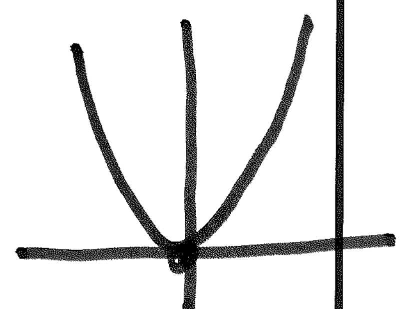
<u>Example 2</u> : $f(x) = x$

$f : \mathbb{R} \longrightarrow \mathbb{R}$



$\nexists M_1$ s.t. $|f(x)| \leq M_1 \quad \forall x \in \mathbb{R}$

f is unbounded. f is neither bounded above or below



Example 3: $f(x) = x^2$

$$f : \mathbb{R} \longrightarrow [0, \infty)$$

f is not bounded, but

$$f(x) \geq 0 \quad \forall x \in \mathbb{R}.$$

M_3

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Therefore f is bounded below.

$\inf f = g.l.b\ f = 0.$

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Sequences

Def: A sequence $S = \{a_n\}_{n=1}^{\infty}$ of real numbers is a function from $\mathbb{N}$ to $\mathbb{R}$.

$$S = \{a_1, a_2, a_3, \ldots\}$$

$$S : \mathbb{N} \longrightarrow \mathbb{R}$$

$$1 \longmapsto a_1, \quad 2 \longmapsto a_2, \quad 3 \longmapsto a_3, \ldots, \quad n \longmapsto a_n \ldots$$

Example :

$$\left\{ \frac{1}{n} \right\}_{n=1}^{\infty} = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \dots \right\}$$

$$\left\{ \left(\frac{1}{2}\right)^n \right\}_{n=1}^{\infty} = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \right\}$$

Extra ①

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$$|5| = 5$$

$$|-19| = 19 = -(-19)$$

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Extra ②

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$$[a, \infty) = \{x \in \mathbb{R} : x \geqslant a\}$$

$$(a, \infty) = \{x \in \mathbb{R} : x > a\}$$

$$(-\infty, b] = \{x : x \leq b\}$$

$$(-\infty, b) = \{x : x < b\}$$