MATH 471

INTRODUCTION TO ANALYSIS I

SESSION no. 4

Last class :

limits of sequences

Today :

Some properties of convergent sequences

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If $\{a_n\}_{n=1}^{\infty}$ such that the limit is a then we write

$$\lim_{n \to \infty} a_n = a.$$

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Def If the seq $\{a_n\}_{n=1}^{\infty}$ has the limit a then $\{a_n\}_{n=1}^{\infty}$ is a convergent sequence, that converges to a.

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If such a limit does not exist then the sequence is said to be divergent.

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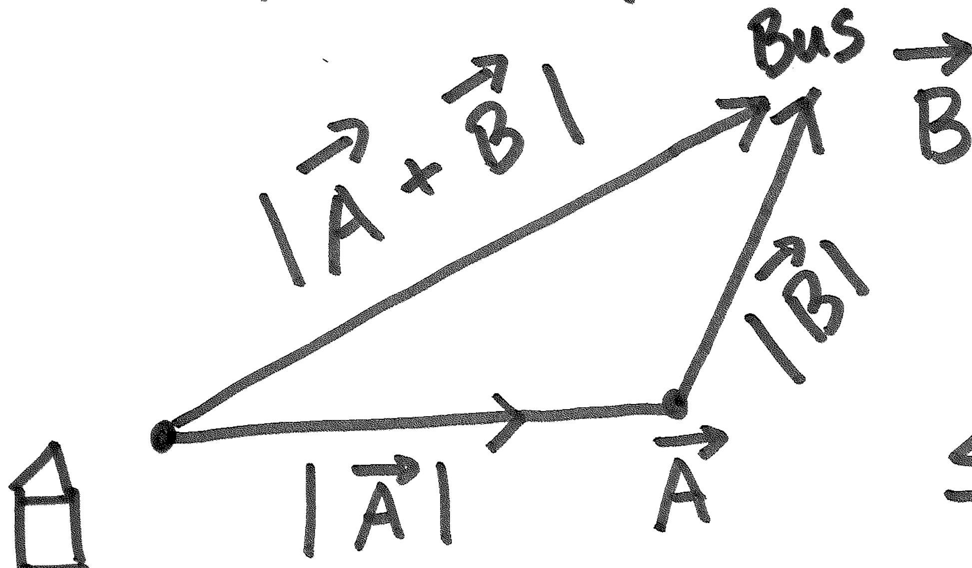
Triangle Inequality

If a & b are two real numbers then

$$|a+b| \leq |a| + |b|$$



$$|\vec{A}+\vec{B}| \leq |\vec{A}| + |\vec{B}|$$

Bounded sequences

Def: A sequence $\{a_n\}$ is bounded if $\exists\, M \in \mathbb{R}$ s.t.

$$|a_n| \leq M, \quad n \in \mathbb{N}.$$

Theorem: If a sequence $\{a_n\}$ is convergent, then it is bounded.

Proof: Let $L = \lim_{n\to\infty} a_n$. Let $\varepsilon = 1$. Then $\exists N \in \mathbb{N}$ s.t.

$$|a_n - L| < 1, \quad n \geq N.$$

$$\begin{aligned} |a_n| &= |a_n - L + L| \\ &\leq |a_n - L| + |L| \quad \text{(by Triangle)} \\ &< 1 + |L|, \quad n \geq N \end{aligned}$$

$$M_1 = \max\{|a_1|, |a_2|, \ldots, |a_{N-1}|\}$$

Then

$$|a_n| \le \underbrace{M_1 + |L| + 1}_{M}, \quad n \in \mathbb{N}$$

Thus $\{a_n\}$ is bounded $\square$

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Converse

Should a bounded sequence be convergent? No

$\{1, -1, 1, -1, \cdots\}$

bounded, $|a_n| \le 1$

divergent sequence

(Try to prove)

A divergent sequence may or may not be bounded.

$\{1, 2, 3, \ldots n, \ldots\}$
divergent, not bounded

$\{1, -1, 1, -1, \ldots\}$
divergent, bounded