MATH 471

INTRODUCTION TO ANALYSIS I

SESSION no. 10

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Theorem

$f : D \longrightarrow \mathbb{R}$, $g : U \longrightarrow \mathbb{R}$
$f(D) \subseteq U$. Suppose that
f is continuous at $x_0 \in D$
and g is continuous at
$f(x_0)$. Then $g \circ f : D \to \mathbb{R}$ is
continuous at x_0.

2 $g \circ f : D \to \mathbb{R}$ $g(f(x))$

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Let $\{x_n\}$ be a sequence in D such that $\lim_{n \to \infty} \{x_n\} = x_0$.

Since f is continuous at x_0, $\{f(x_n)\}$ converges to $f(x_0)$.

Since g is continuous at $f(x_0)$, $\{g(f(x_n))\}$ converges to $g(f(x_0))$.

[Note $\{f(x_n)\}$ is a seq in U]

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Therefore for any $\{x_n\} \in D$

$\{g \circ f(x_n)\}$ converges to

$(g \circ f)(x_0)$

$\Rightarrow$ $g \circ f$ is continuous at x_0

$\square$

Closed sets

$$(a,b)=\{x\in\mathbb{R}: a<x<b\}$$

NOT CLOSED

$$\left\{a+\frac{1}{n}\right\}_{n=1}^{\infty} \in (a,b)$$

$$\downarrow$$

$$a \notin (a,b)$$

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NOT CLOSED

$$[a, b) = \{a \leq x < b\}$$

$$\left\{a + \frac{1}{n}\right\}_{n=1}^{\infty} \longrightarrow a \in [a, b)$$

$$\left\{b - \frac{1}{n}\right\}_{n=1}^{\infty} \longrightarrow b \notin [a, b)$$

$$[a, b] = \{a \leq x \leq b\}$$

For any sequence $\{x_n\} \in [a,b]$ that converges to some c, $c \in [a,b]$.

$[a,b]$ is a <u>closed</u> and bounded interval.

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Closed set

A set S is said to be **closed** if $\{a_n\}$ is a sequence in S that converges to a, then the limit a also belongs to S.

NOT CLOSED

$\mathbb{Q}$ - set of rationals in $\mathbb{R}$.

Let $a = \sqrt{2}$.

$\{1, 1.4, 1.41, 1.414, \dots\}$
is a sequence in $\mathbb{Q}$.
Converges to $a = \sqrt{2} \notin \mathbb{Q}$.

Definition

A function $f : D \to \mathbb{R}$ attains a maximum if $\exists$ a point $x_0 \in D$ s.t.

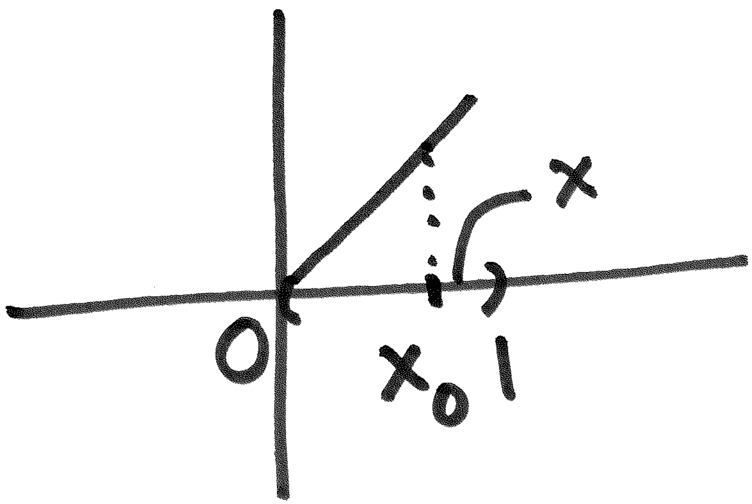
$$f(x) \leq f(x_0), \ \forall x \in D,$$

attains a minimum if $\exists$ a point $x_0 \in D$ s.t

$$f(x) \geq f(x_0), \ \forall x \in D.$$

Example

$f : (0,1) \longrightarrow \mathbb{R}, \quad f(x) = x$



If x_0 is chosen then for $x \in (x_0, 1)$

$f(x) > f(x_0)$

Can't pick 1 since $1 \notin (0,1)$.

f does not attain a max.

f also does not attain a min.

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$f : [0,1] \longrightarrow \mathbb{R}$

$f(x_0) = f(1)$
$= 1$

If $x_0 = 1$, then

$f(x) \le 1 = f(x_0)$, $\forall x \in [0,1]$

f attains a max. at $x = 1$.

If $x_0 = 0$, then

$f(x) \geq f(0) = 0$, $\forall x \in [0,1]$

f attains a min at $x = 0$.

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Next class: Extreme Value Thm:

If f is continuous on a closed and bounded set then f attains a max & a min.