MATH 471

INTRODUCTION TO ANALYSIS I

SESSION no. 24

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Applications of the mean value theorem.

2

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Criteria for monotonicity

1) Last class : $f'(x) > 0 \Rightarrow f$ is strictly increasing

Similarly : If $f'(x) < 0$ then f is strictly decreasing

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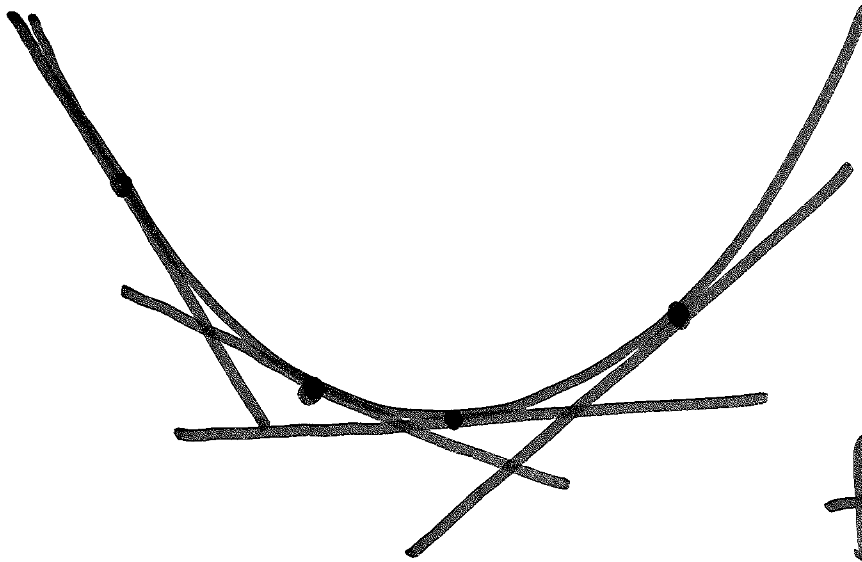
Concave function



f' is increasing from left to right

$\Rightarrow f'' > 0$

Graph is always above the tangent lines

Such a function is said to be concave upward.

4

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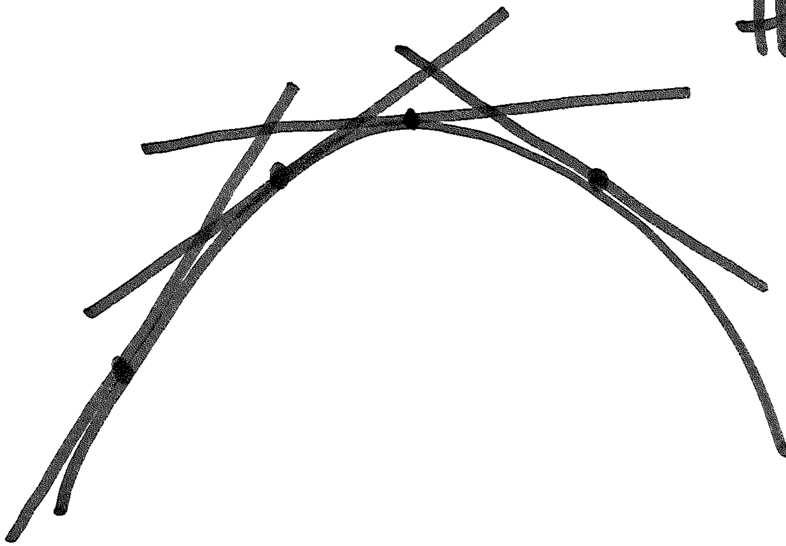
As we go from left to right the slope of the tangent lines change from positive to negative



$\Rightarrow f'$ is decreasing

$\Rightarrow f'' < 0$

Graph of f is always below the tangents

Such a graph is concave downwards

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Theorem

If f satisfies $f''(x) > 0$ for all $x \in (a, b)$ then f is ~~is~~ concave upward on (a,b)

Similarly, if $f''(x) < 0$ on (a,b) then f is concave downward.

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Point of inflection: A point in the domain of f at which f changes its concavity (changes from concave down to up or vice-versa)

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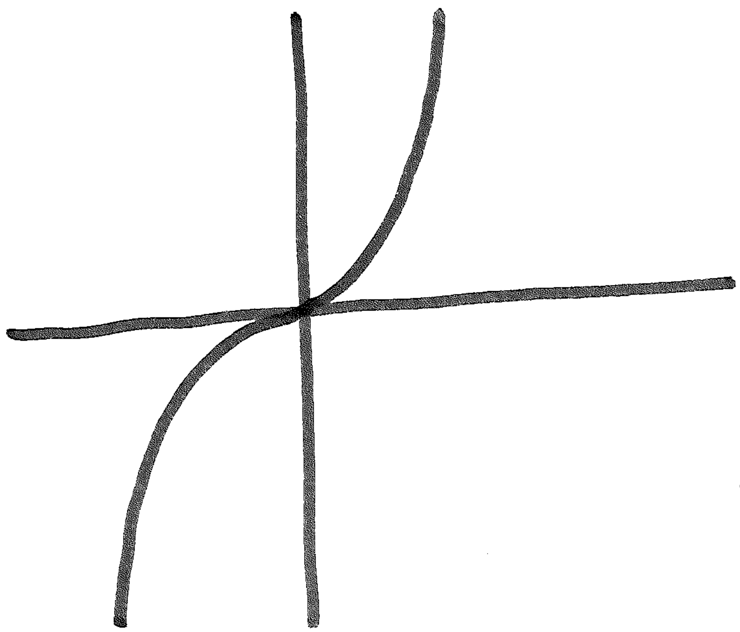
Example on pt. of inflection

$f(x) = x^3$

$x = 0$ is the point of inflection.



For $x < 0$: Concave down

$x > 0$: concave up.

$f'(x) = 3x^2$, $f''(x) = 6x = 0$ @ $x = 0$

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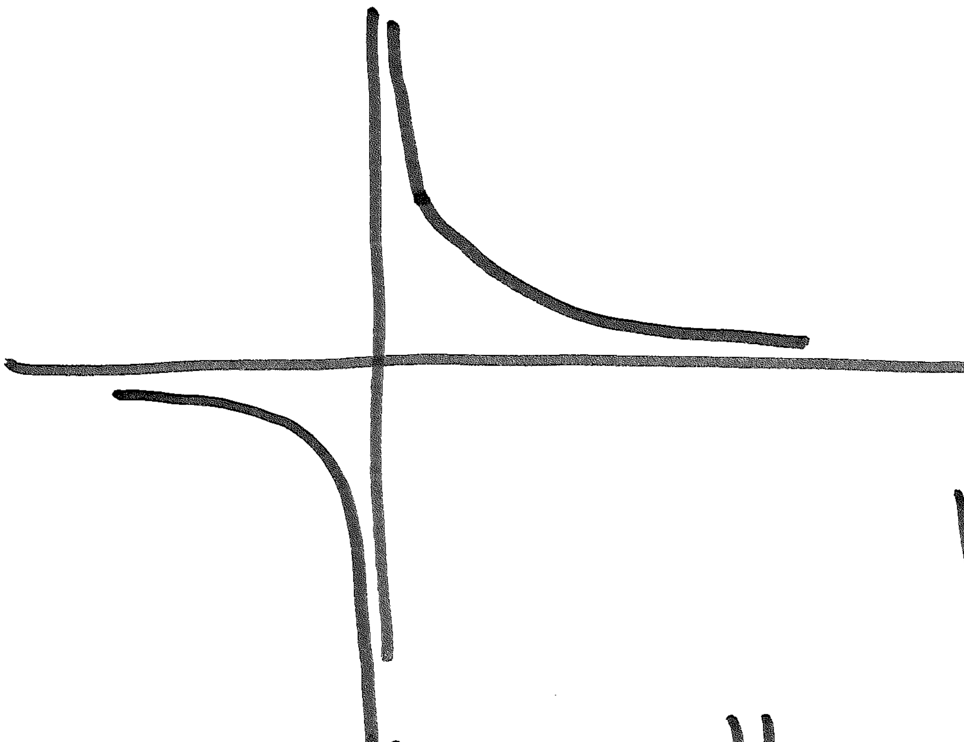
Example.

$f(x) = \frac{1}{x}$ Domain : $\mathbb{R} \setminus \{0\}$



f changes Concavity at $x = 0$ but since 0 is not in the domain it is not a point of inflection.

9

possibly 2 lectures back

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Recall: $f:(a,b) \to \mathbb{R}$ is differentiable and $x_0 \in (a,b)$ is a local extrema then $f'(x_0) = 0$.

Now, we want to know if x_0 is a local max. or local min.

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Theorem

Suppose $f: (a,b) \to \mathbb{R}$ is s.t. f'' exists. Suppose that $f'(x_0) = 0$

(a) If $f''(x_0) > 0$, then x_0 is a local min

(b) If $f''(x_0) < 0$, then x_0 is a local max

Only (a) is proved. Proof of (b) is similar to (a).

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(a) Let $f''(x_0) > 0$. Then

$$f''(x_0) = \lim_{x \to x_0} \frac{f'(x) - \overset{\nearrow 0}{f'(x_0)}}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{f'(x)}{x - x_0} > 0$$

$$\Rightarrow \begin{cases} f'(x) > 0 & \text{if } x \in (x_0, x_0 + \delta) \\ f'(x) < 0 & \text{if } x \in (x_0 - \delta, x_0) \end{cases}$$

12

$x_0 \quad x \quad x_0+\delta$

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Let $x \in (x_0, x_0+\delta)$: By the MVT

$$f(x) - f(x_0) = f'(c)(x - x_0)$$

for some $c \in (x_0, x)$.

$f'(c) > 0$ & $(x - x_0) > 0$

and so $f(x) - f(x_0) > 0$

$\Rightarrow f(x) > f(x_0)$ if $x \in (x_0, x_0+\delta)$.

13

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Taking $x \in (x_0 - \delta, x_0)$, Similarly,

$f(x) < f(x_0)$, $x \in (x_0 - \delta, x_0)$

Therefore,

$f(x) < f(x_0)$, $x \in (x_0 - \delta, x_0 + \delta)$

$\Rightarrow$ ~~f~~ x_0 is a local minima.

$\square$