MATH 472

INTRODUCTION TO ANALYSIS II

SESSION no. 41

University of Idaho **Problems**

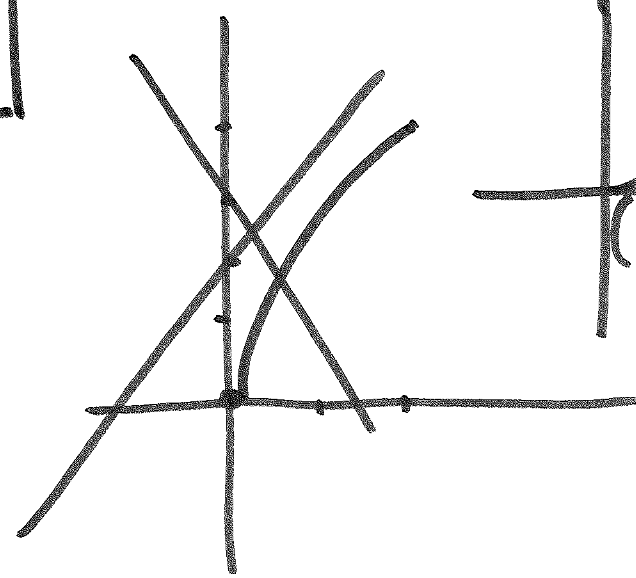
1 b) $\displaystyle\int_C xy\,ds$ C is the parabola $y = x^2$ between $(0,0)$ & $(2,4)$

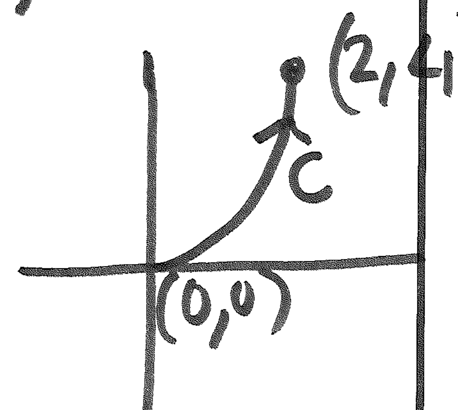
$x(t) = t$

$y(t) = t^2$ $\boxed{0 \le t \le 2}$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \sqrt{1 + 4t^2}\,dt$$





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$$\int_0^2 t\, t^2 \sqrt{1+4t^2}\, dt$$

$$= \int_0^2 t^3 \sqrt{1+4t^2}\, dt$$

$$= \int_0^2 \underbrace{t^2}\, \underbrace{t\sqrt{1+4t^2}\, dt}$$

then use integration by parts

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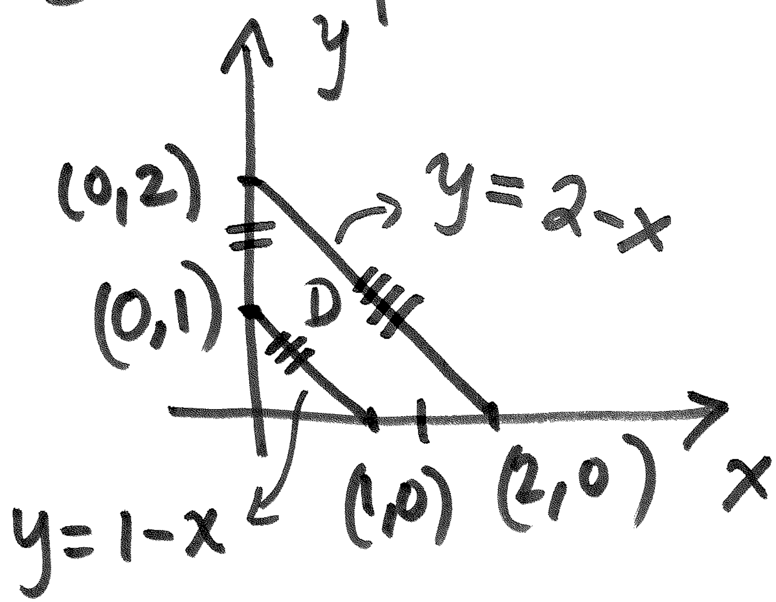
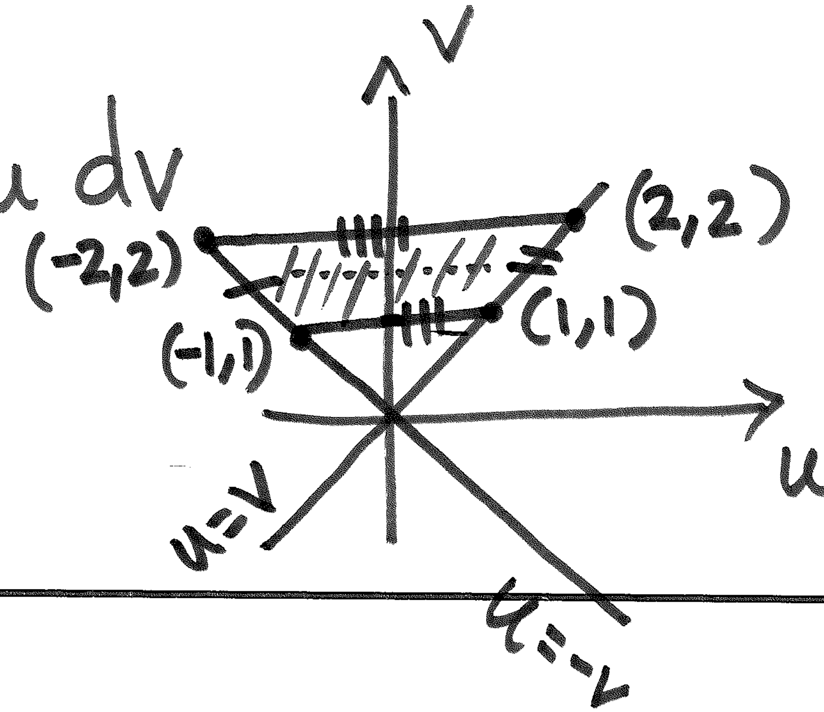
5 $$\iint_D \sin\left(\frac{y-x}{y+x}\right)$$

D : trapezoid



$$y - x = u$$
$$y + x = v$$

$$\int_{\boxed{?}}^{\boxed{?}}\int_{\boxed{?}}^{\boxed{?}} \sin\left(\frac{u}{v}\right) |J| \, du \, dv$$



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① $y=0$ $\quad \left.\begin{array}{l} u = -x \\ v = x \end{array}\right\} \Rightarrow u = -v$

$1 \le x \le 2 \Rightarrow 1 \le v \le 2$

② $x=0$ $\quad \left.\begin{array}{l} y = u \\ y = v \end{array}\right\} \Rightarrow u = v$

$1 \le y \le 2 \Rightarrow 1 \le u, v \le 2$

③ $y = 1-x$ $\quad u = 1-2x \quad 0 \le x \le 1 \Rightarrow$

$v = 1 \quad -1 \le u \le 1$

④ $y = 2-x$ $\quad u = 2-2x \quad 0 \le x \le 2$

$v = 2 \quad \Downarrow$

$-2 \le u \le 2$

$$y = \frac{u+v}{2} = h(u,v), \quad x = \frac{v-u}{2} = g(u,v)$$

$$J = \begin{bmatrix} \partial g/\partial u & \partial g/\partial v \\ \partial h/\partial u & \partial h/\partial v \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

$$|J| = \left|\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right| = \left|-\frac{1}{2}\right|$$

$$= \frac{1}{2}$$

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$$\int_1^2 \int_{-v}^{v} \sin\left(\frac{u}{v}\right) \frac{1}{2} \, du \, dv$$

$\vdots$

#3, #7 Find the potential of a given $\vec{F}$ means find f s.t. $\nabla f = \vec{F}$

gradient

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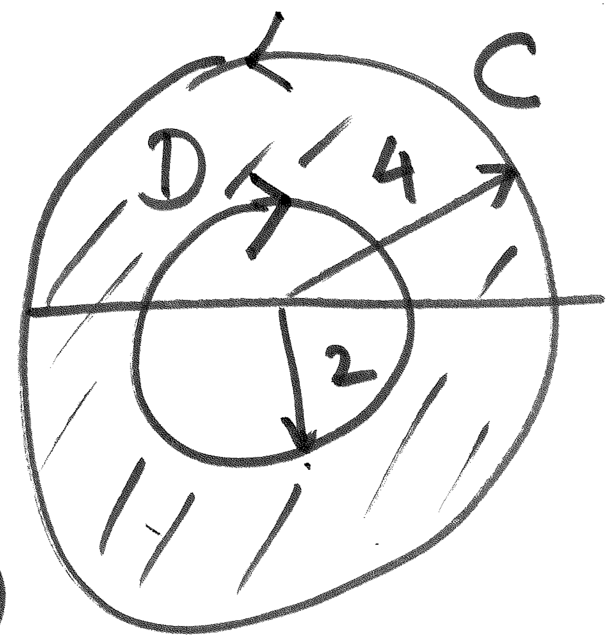
#6 $x^2+y^2=4$, $x^2+y^2=16$



C: boundary is positively oriented [keep D to your left)

$\vec{F} = (-xy, x)$

$$\int_C \vec{F} \cdot dr = \int_C M\,dx + N\,dy \overset{\text{Green's Thm}}{=} \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$$

$M = -xy$ $M_y = -x$

$N = x$ $N_x = 1$

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$$= \iint_D (1+x)\,dx\,dy \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$= \int_0^{2\pi}\int_2^4 (1+r\cos\theta)\,\underbrace{r\,dr\,d\theta}_{dx\,dy}$$

$$\vdots$$

$$= 12\pi$$