Population Growth Models: Covariates and Viability Analyses

Managers/Researchers often find that population trend data from time series don't perfectly fit the growth models that we've studied so far.

Or are interested in how additional environmental factors influence population trends.

These other factors, or sources of additional variation, can be accounted for by adding them to the model as covariates

Recall the formula for stochastic logistic population growth:

$$\ln(n_{t+1}) = \ln(n_t) + r_{max} + b(n_t) + F + \epsilon$$

Case study: San Joaquin Kit Foxes in California

Dennis & Otten (2000)

Endangered population

Highly variable, fluctuating population

Highly variable environmental conditions, with potential time lag in response to rainfall

Coyote predation could be a concern

Goals:

Find the best model explaining SJ Kit Fox numbers over time, including several covariates

Use that model to predict future numbers and conduct a population viability analysis

General model of population growth:

$$N_t = N_{t-1}\exp(a + bN_{t-1} + cW_{t-2} + \sigma Z_t), \quad (1)$$

Where:
a = r_{max}

b = strength of density dependent relationship

c = coefficient for rainfall covariate

W_{t-2} = growing season rainfall 2 yrs previous (time lag)

σ = coefficient for Process Noise (F)

$$\ln(n_{t+1}) = \ln(n_t) + r_{max} + b(n_t) + c\,W_{t-2} + F$$

$\ln(n_{t+1}) = \ln(n_t) + r_{max} + b(n_t) + c\,W_{t-2} + F$

If this is the general model, what models are tested in table below?

Table 1. Maximum likelihood estimates ($\hat{a}, \hat{b}, \hat{c}, \hat{\sigma}^2$) of parameters in the density dependence–rainfall model (Eq. 1), generalized R^2, and Schwarz information criterion (SIC) for 4 model hypotheses (H_i) fitted to the San Joaquin kit fox data. The population estimates (and SE) for the winters of 1983–84 to 1995–96 were 117 (11.5), 220 (16.6), 161 (15.9), 164 (14.3), 135 (13.4), 166 (14.7), 131 (13.9), 117 (13.0), 46 (9.2), 88 (9.2), 196 (15.9), 363 (23.5), and 133 (11.0); the precipitation levels (cm) for the growing seasons of 1982–83 to 1993–94 were 22.0, 8.6, 8.7, 15.2, 12.9, 10.2, 8.4, 5.9, 14.8, 17.6, 22.5, and 11.3, respectively.

Hypothesis	$\hat{a}$	$\hat{b}$	$\hat{c}$	$\hat{\sigma}^2$	R^2	SIC
H_0	0.01068	—	—	0.3305	0.00	25.7
H_1	0.7408	−0.004647	—	0.2055	0.09	22.5
H_2	−1.089	—	0.08346	0.1446	0.23	18.3
H_3	−0.3607	−0.003835	0.07437	0.06165	0.82	10.6

a, b, c-hats are parameter estimates from data.

a blank is equivalent to making that parameter value = 0…..what does this do?

can you explain/describe the models above (variations on growth models we’ve seen)

$$H_0: \ln(n_{t+1}) = \ln(n_t) + r_{max} + b(n_t) + c\,W_{t-2} + F$$

$\downarrow$

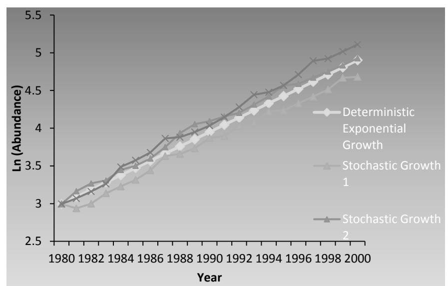
$$H_0: \ln(n_{t+1}) = \ln(n_t) + r_{max} + 0(n_t) + 0\,W_{t-2} + F$$

$\downarrow$

$$H_0: \ln(n_{t+1}) = \ln(n_t) + r_{max} + F$$

What pop. growth model is this?

Stochastic Exponential Growth with Process Noise



What about the others?

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H_0 : Stochastic Exponential Growth w/ Process Noise (F)

H_1 : Stochastic Density Dependent Growth w/ Process Noise (F)

H_2 : Stochastic Exponential Growth w/ Rain Covariate and Process Noise (F)

H_3 : Stochastic Density Dependent Growth w/ Rain Covariate and Process Noise (F)

Which hypothesis/model is supported?

Used this model (H_3) to predict population size at next time step.

Did a pretty good job
$R^2 = 0.82$

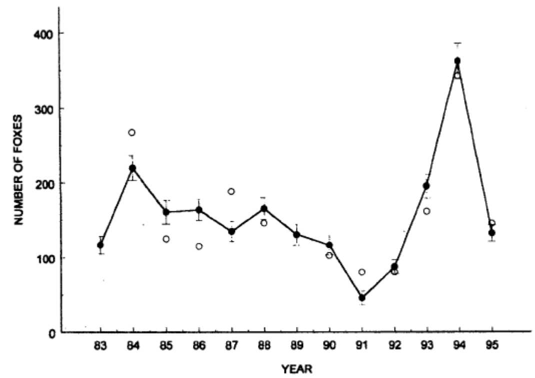

Fig. 1. Estimates of San Joaquin kit fox abundances (solid circles, ±SE) and 1-step-ahead predictions (open circles) from fitted model of density-dependent population growth with growing season rainfall as a covariate.

Do coyotes matter to SJ Kit Fox numbers?

$$N_t = N_{t-1}\exp(a + bN_{t-1} + c_1W_{t-2} + c_2V_{t-1} + \sigma Z_t), \qquad (2)$$

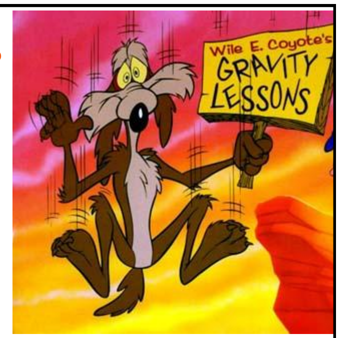

Table 2. Maximum likelihood estimates ($\hat{a}$, $\hat{b}$, $\hat{c}_1$, $\hat{c}_2$, $\hat{\sigma}^2$) of parameters in the density dependence–rainfall–coyote model (Eq. 2), and Schwarz information criterion (SIC) for 7 model hypotheses (H_i') fitted to the San Joaquin kit fox data, with growing season rainfall (2-yr lag) and coyote abundance index (1-yr lag) as covariates (coefficients c_1 and c_2, respectively). The coyote abundance index values for March 1985 to March 1995 were 110.5, 135.0, 106.06, 45.0, 36.27, 30.15, 25.13, 21.86, 68.42, 100.5, and 116.16, respectively.

Hypothesis	$\hat{a}$	$\hat{b}$	$\hat{c}_1$	$\hat{c}_2$	$\hat{\sigma}^2$	R^2	SIC
H_0'	−0.04575	—	—	—	0.3223	0.00	23.6
H_1'	0.6625	−0.004360	—	—	0.2055	0.11	21.0
H_2'	−1.137	—	0.08819	—	0.1559	0.12	18.0
H_3'	−0.06824	—	—	−0.001577	0.3183	0.00	25.8
H_4'	−0.9531	—	0.09333	−0.003424	0.1375	0.40	19.0
H_5'	0.5396	−0.006478	—	0.006462	0.1653	0.19	21.0
H_6'	−0.4255	−0.003924	0.08223	—	0.06206	0.84	10.2
H_7'	−0.4019	−0.004969	0.07601	0.003085	0.05372	0.85	11.0