Mark-Recapture

Useful in estimating:
- a. Size of population
- b. Rate of exploitation
- c. Survival rate
- d. Rate of recruitment

Lincoln-Petersen Estimate

- Applicable to closed populations
- A sample is taken, marked and released back into the population = M
- A second sample is taken of n individuals of which m have marks

Lincoln-Petersen Estimate

- Proportion in sample marked = m/n
- This proportion should be equal to the proportion of marked population (M) to total population (N)
- = M/N
- e.g. M/N = m/n

Chapman’s modification

- \((M+1)(n+1)\)
- \(N_{\text{est}} = \) \(\frac{(m+1)}{n-m}\)
- \(N_{\text{est}}\)\(^2\) = \(n+1)(m+2)\)

Rate of Exploitation

- Where the second sample is taken in course of harvesting the population (much of fisheries, waterfowl, control efforts for pests)
- Rate of exploitation \((u)\)
- \(u_{\text{est}} = \frac{m}{M}\)
Assumptions:
1. Population is closed (no births or deaths or movements out or in)
2. All animals have same probability of being caught in the first sample
3. Marking does not affect catchability of an animal
4. Second sample is a random sample

5. No loss of tags between samples
6. All tags are reported in the second sample

Multiple Mark-Recaptures
Assuming a closed population, Schnabel (1938) and Schumacher & Eschmeyers (1943) developed
\[ N_k = \frac{E n_i M_i}{(E m_i) + 1} \]
Note: All sums from i=2 to k

Probability of Capture
“Probability of capture is equal and constant for each animal at each trapping occasion.”
Problems:
- Day to day variation (weather) = time
- Behavioral effects (trap happy/shy)
- Individual differences (heterogeneity)

Capture Program
Models developed to handle these problems based on maximum likelihood (ML) in 50’s, 60’s, 70’s.
Not applied until 1980’s because of difficulty of calculations.

CAPTURE Program: Models
- \( M_o \) Constant capture probabilities
- \( M_t \) Variation by time = Schnabel
- \( M_b \) Behavioral response to trapping
- \( M_{bh} \) Behavior and heterogeneity
- \( M_{tb}, M_{tb}, M_{tbh} \)

CAPTURE Program
Key requirement is to mark animals individually so that their full capture history can be recorded.
- Numbers vs. density
- Boundary problems
Trapping Web

- Standard approach is to lay out traps in a rectangular grid (See CAPTURE concentric rows of traps)
- Record location of initial capture of each animal.
- Density of captures in centermost circles estimates density using variable circular plot approach.

Open Population

- Limitation of previous methods is assumption of closure (no births, deaths, immigration or emigration).
- Can we estimate for open populations?
- What problem does mortality cause?
- Marked population is unknown because some of these have died.

Open Population

- Jolly (1965) - English statistician and
- Seber (1965) - New Zeland statistician
- Independently developed solution for multiple mark-recapture study based on earlier work by:
  - Darroch (1959) another English statistician
  - Cormack (1964) a Scottish statistician.

Jolly-Seber Model (Often called Cormack-Jolly-Seber)

- Use results of sampling at later time periods to estimate how many of the marked animals were present at an earlier time period.
- To do this we must give each animal an individual mark so that its entire capture history can be recorded.

Capture Recapture Totals

<table>
<thead>
<tr>
<th>Time</th>
<th>Captured</th>
<th>Recaptures</th>
<th>Released</th>
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<td>10</td>
<td>143</td>
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<tr>
<td>3</td>
<td>169</td>
<td>37</td>
<td>164</td>
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<td>209</td>
<td>56</td>
<td>202</td>
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<td>5</td>
<td>220</td>
<td>53</td>
<td>214</td>
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<tr>
<td>6</td>
<td>209</td>
<td>77</td>
<td>207</td>
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Recapture Matrix

<table>
<thead>
<tr>
<th>Time of Capture</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>Time of Last Capture</td>
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<td>1</td>
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<tr>
<td>2</td>
<td>34</td>
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<td>5</td>
<td>43</td>
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JS Population Estimate

- \[ n_i M_i \]
- \[ N_i = \]
- \[ m_i \]
- If we don’t actually know \( M_i \), we can use an estimate of \( M_i \).
**JS Survival Estimate**
- \( M_{i+1} \)
- \( s_i = \frac{M_i + n_i - m_i}{M_i + N_i s_i} \)

**JS Estimate of Births**
- \( N_{i+1} = \text{Additions} + \text{Survivors from } N_i \)
- \( = B_i + N_i s_i \)
- Rearranging this for births
- \( B_i = N_{i+1} - N_i s_i \)

**JS Marked Population**
- How do we estimate the size of the marked population?
- We have to do a mark-recapture estimate of that.
- Use the animals not seen at the first recapture sample but seen later on as our recaptures.
- We do a mark-recapture within a mark-recapture estimate.

**JS Marked Population**
- Which of the rest are known to be alive?
- Some of the rest are caught after sample \( i \)
- Call these \( z_i \)
- \( z_i = \text{Animals marked previous to sample } i, \text{not caught at } i, \text{but caught later.} \)

**JS Marked Population**
- \( n_i \) is largest group of individuals known to be alive at sample \( i \) and it is comparable to \( (M_i - m_i) \), the “rest”
- Denote by \( r_i \), the number of \( n_i \) observed after sample \( i \).
- \( r_i \) is some fraction of \( n_i \)

**Recapture Matrix**
<table>
<thead>
<tr>
<th>Time of Capture</th>
<th>1</th>
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<td>43</td>
</tr>
</tbody>
</table>
Marked Population at t=3

- \( r_3 = 33 + 13 + 8 = 54 \)
- \( z_3 = 5 + 2 + 2 + 18 + 8 + 4 = 39 \)
- \( n_3 = 169 \)
- \( m_3 = 37 \)

\( z \times n = 39 \times 169 \)

\( M_3 = \frac{r_3}{m_3} + m_3 = \frac{54}{37} + 37 = 155.5 \)

Population at t=3,4

- \( M_4 = \frac{202 \times 37}{50} + 56 = 205.5 \)
- \( N_3 = n_3 \times M_3 / m_3 = 169 \times 155.5 / 37 = 710 \)
- \( N_4 = 209 \times 205.5 / 56 = 767 \)
- \( S_3 = M_4 / (M_3 + n_3 - m_3) = 205.5 / (155 + 169 - 37) = 0.72 \)
- \( B_4 = N_4 - N_3 \times S_3 = 767 - 710 \times 0.72 = 256 \)

Upshot?

- For this real example we can estimate population size, at each sample except last, as well as birth rate and death rate between all surveys except last.
- Variance of each estimate and its standard error can be calculated in MARK or JOLLY software.
- Requires large numbers of marked animals and recaptures for decent estimates.

Closed vs. Open Estimators?

- Closed (CAPTURE) allows us to test more of the assumptions and estimate correctly even if some assumptions aren't met.
- Open (MARK) isn't biased by lack of closure but can't deal with all the problems that closed estimators handle.
- Could they be combined?

Combination of Open and Closed Models

= Pollock’s Robust Design

- Ken Pollock (prof at North Carolina State Univ.) developed a clever, robust design in 1982
- Involves methods assuming closed popn during closely spaced multiple recapture samples
- Applies Cormack-Jolly-Seber methods for open popn during wider intervals between intense samples above
- MARK by Gary White incorporates this combined approach, but now you really need lots of data!