Problem Set 15

Problem 1. Let $H$ be a hexagon. The intersections of the opposite sides of $H$ determine 3 points $A, B, C$. Prove that if $A, B, C$ lie on a line then the vertices of the hexagon lie on a conic.

Problem 2. Let $C$ be an irreducible curve of degree 2 in $\mathbb{P}^2$. Prove that the dual of $C$ is a conic.

Problem 3. State the dual form of Pascal’s theorem.

Problem 4. State the dual form of Pappus’s theorem.

Problem 5. Give a brief explanation of why there is not a conic passing through 6 general points in $\mathbb{P}^2$.

Problem 6. Let $P_1, P_2, \ldots, P_6$ be 6 points on an irreducible curve, $C$, of degree 2 in $\mathbb{P}^2$. Let $L_{ij}$ denote the line passing through points $P_i$ and $P_j$. Let $Q_1 = L_{12} \cap L_{34}, Q_2 = L_{23} \cap L_{45}, Q_3 = L_{34} \cap L_{56}, Q_4 = L_{45} \cap L_{61}, Q_5 = L_{56} \cap L_{12}, Q_6 = L_{16} \cap L_{23}$. Prove that there is a conic passing through $Q_1, \ldots, Q_6$.

Problem 7. Let $C$ and $D$ be curves in $\mathbb{P}^2$ of degree $n$. Suppose $C$ and $D$ meet in exactly $n^2$ points. Suppose there is a curve of degree $m < n$ which contains $mn$ of the $n^2$ points. Use Bezout’s theorem to prove that there is a curve $F$ of degree $n - m$ which contains the remaining $n^2 - mn$ points.