Homework 5 (Math461 EO)

Problem 1 (2.5 points)

Let S_3 be the symmetric group of degree 3, i.e, the group of permutations on $\{1, 2, 3\}$.

(i) Find the order of each element of S_3.

(ii) List all cyclic subgroups of S_3.

Problem 2 (2.5 points)

Use trigonometric identities and induction to prove that

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$$

for all $n \in \mathbb{N}$. Show that for a constant θ

$$H = \left\{ \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix} \middle| n \in \mathbb{Z} \right\}$$

is a cyclic subgroup of $GL(n, \mathbb{R})$. Do you think H is finite?

Problem 3 (2.5 points)

Let a be an element of a group G and let $|a| = 15$. Compute the orders of the following elements of G:

(i) a^3, a^6, a^9 and a^{12}.

(ii) a^5 and a^{10}.

(iii) a^2, a^4, a^8 and a^{14}.

Problem 4 (2.5 points)

Let G be a group with respect to addition and let $a \in G$. Prove that $|a| = |-a|$.