

Homework 7 (Math461 EO)

Problem 1 (4 points)

Each of the following rules determines a map $\phi : \mathbb{R}^\times \rightarrow \mathbb{R}^\times$. Decide in each case whether or not ϕ is a homomorphism. For those that are homomorphisms, state the kernel and decide whether ϕ is onto.

(i) $\phi(x) = |x|$.

(ii) $\phi(x) = \frac{1}{x}$.

(iii) $\phi(x) = \frac{|x|}{x}$.

(iv) $\phi(x) = -x$.

Problem 2 (3 points)

Consider the additive group \mathbb{Z}_{12} and define $\phi : \mathbb{Z}_{12} \rightarrow \mathbb{Z}_{12}$ by $\phi([x]) = [3x]$. Prove that ϕ is a homomorphism and find $\ker(\phi)$.

Problem 3 (3 points)

Let G be a group with respect to binary operation $*$. If a is a fixed element of G , define $\gamma_a : G \rightarrow G$ by $\gamma_a(x) = a * x * a^{-1}$ for all $x \in G$. Prove that γ_a is an isomorphism.