# Homework 8 (Math461 EO)

### Problem 1 (2 points)

Show that

$$H = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$$

is a normal subgroup of $GL(2, \mathbb{R})$.

### Problem 2 (2 points)

Let $G$ be an arbitrary group. Prove that if $H$ and $K$ are normal subgroups of $G$, then $H \cap K$ is also a normal subgroup.

### Problem 3 (3 points)

Let $G$ be a group. For an arbitrary subgroup $H$ of $G$, the *normalizer* of $H$ in $G$ is the set $N(H) = \{x \in G \mid xHx^{-1} = H\}$.

(i) Prove that $N(H)$ is a subgroup of $G$.

(ii) Prove that $H$ is a normal subgroup of $N(H)$.

(iii) Prove that if $K$ is a subgroup of $G$ that contains $H$ as a normal subgroup, then $K \subseteq N(H)$.

### Problem 4 (3 points)

Let $A_4$ be the alternating group of degree 4. Assume that

$$H = \{e, (12)(34), (13)(24), (14)(23)\}$$

is a normal subgroup of $A_4$. Write out the distinct elements of $A_4/H$ and make a multiplication table for $A_4/H$.