Homework 9 (Math461 EO)

Problem 1 (4 points)

Let G be an arbitrary group, let H be a subgroup of G and let K be a normal subgroup of G.

(i) Let $HK = \{hk \in G \mid h \in H \text{ and } k \in K\}$. Prove that $HK \leq G$.

(ii) Prove that $H \cap K$ is a normal subgroup of H.

(iii) Prove that K is a normal subgroup of HK.

(iv) Prove that $H/(H \cap K)$ is isomorphic to HK/K.

Hint. Prove that the map $\phi : H \to HK/K$ defined by $\phi(h) = hK$ is an onto homomorphism and then $\ker(\phi) = H \cap K$.

Problem 2 (4 points)

Suppose that ϕ be an onto homomorphism from a group G to another group G'. Let H be a normal subgroup of G that contains $\ker(\phi)$ and let $H' = \phi(H) = \{\phi(h) \mid h \in H\}$.

(i) Prove that H' is a normal subgroup of G'.

(ii) Prove that G/H is isomorphic to G'/H'.

Problem 3 (2 points)

Let $\phi : \mathbb{Z}/8\mathbb{Z} \to \mathbb{Z}/4\mathbb{Z}$ by $\phi(a+8\mathbb{Z}) = a+4\mathbb{Z}$. Show that this is a well-defined, onto homomorphism and describe the kernel explicitly.