Homework 8 (Math462)

Problem 1 (3 points)

Determine which of the polynomials below is (or are) irreducible over $\mathbb{Q}$.

(i) $x^4 + x + 1$

(ii) $x^5 + 5x^2 + 1$

(iii) $x^3 + 6x^2 + 3x + 3$

Problem 2 (4 points)

Let $p(x)$ be an irreducible polynomial over a field F. Suppose that $p(x)$ divides $f_1(x) \cdots f_n(x)$ in $F[x]$. Prove that $p(x)$ divides $f_i(x)$ for some $i \in \{1, \ldots, n\}$.

Problem 3 (3 points)

Let $f(x) = \sum_{i=0}^{n} a_i x^i \in \mathbb{Z}[x]$ with $a_0 \neq 0$ be a monic polynomial. Suppose that $f(x)$ has a zero in $\mathbb{Q}$. Prove that $f(x)$ has a zero m in $\mathbb{Z}$ and that m must divide a_0.

Hint. Use Gauss's lemma whose proof says that if there are polynomials $g(x)$ and $h(x)$ in $\mathbb{Q}[x]$ with $\deg g(x), \deg h(x) < \deg f(x)$ such that $f(x) = g(x)h(x)$, then there are polynomials $G(x)$ and $H(x)$ in $\mathbb{Z}[x]$ such that $\deg G(x) = \deg g(x)$, $\deg H(x) = \deg h(x)$ and $f(x) = G(x)H(x)$.