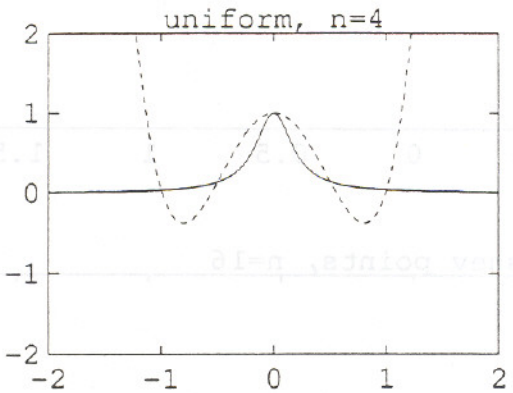
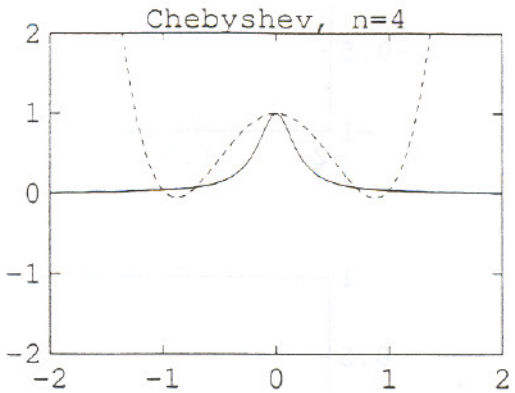
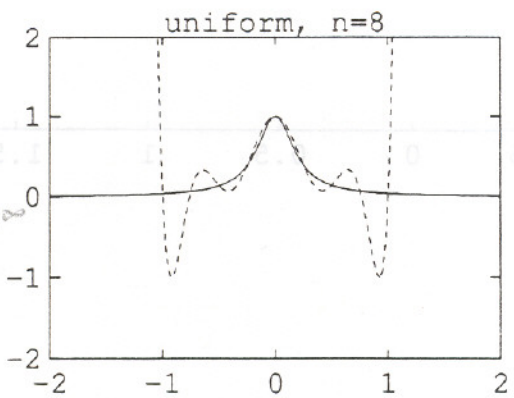
polynomial interpolation

$$f(x) = \frac{1}{1+25x^2} \quad , \quad [a,b] = [-1,1]$$

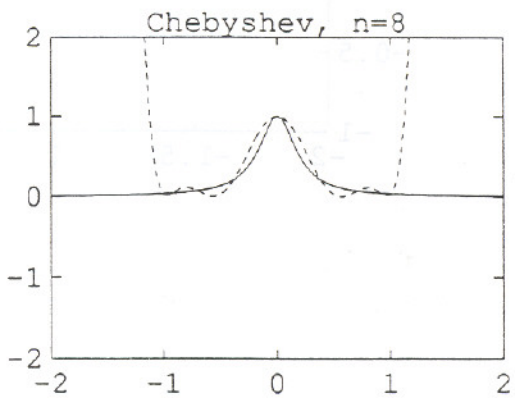
In general, if $f(x)$ and $f'(x)$ are cts on $[-1,1]$, Chebyshev interp. will produce a sequence of polynomials $\{p_n\}$ that converges uniformly to $f(x)$ on $[-1,1]$



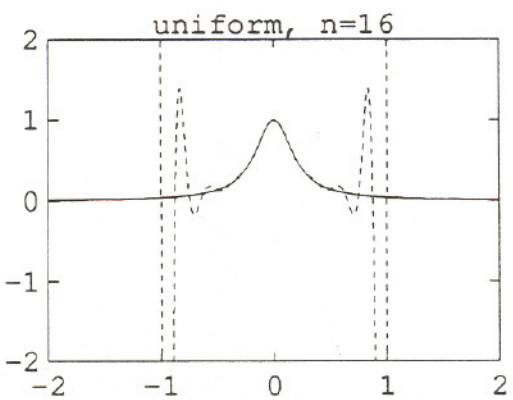


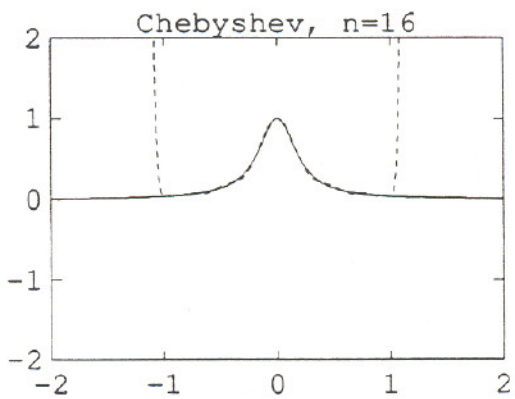


Runge phenomenon: error ↗ as N→∞



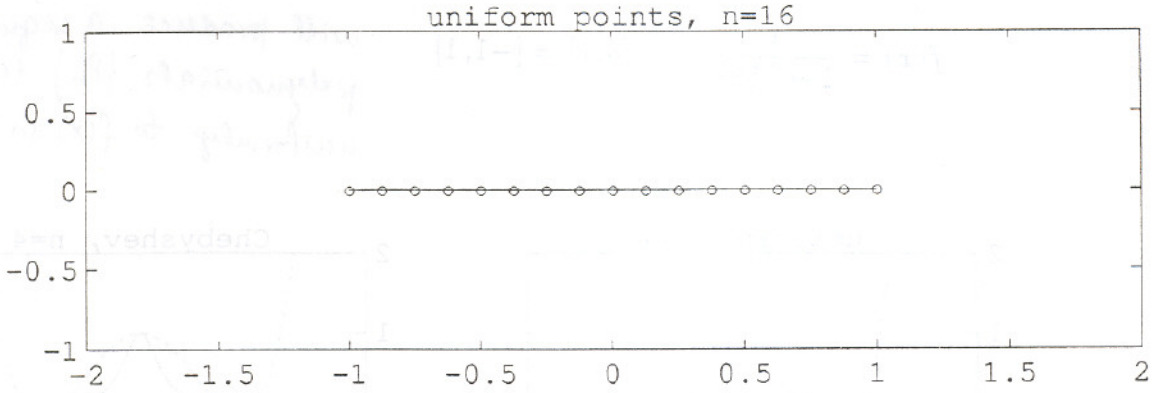
error → 0 as N→∞

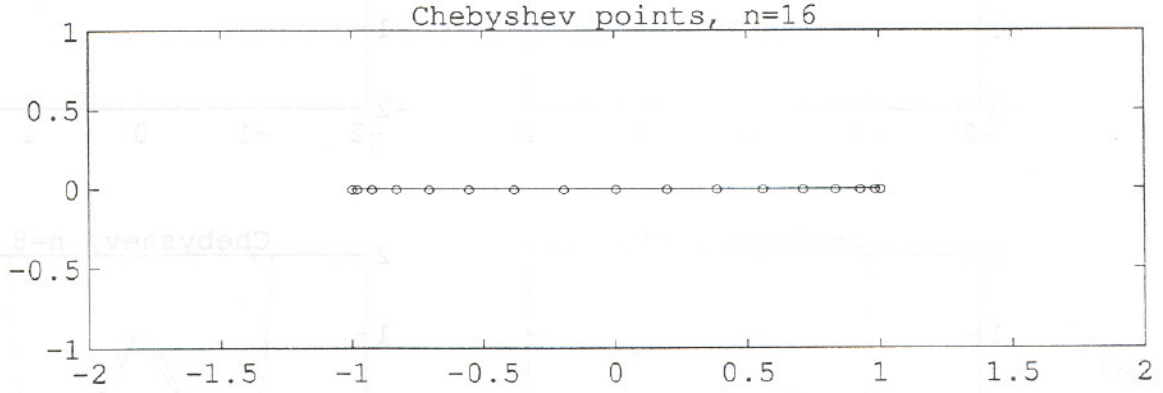




Note:

1. Interpolation at the uniform points is well behaved near the center of the interval but is badly behaved near the endpoints of the interval.
2. Interpolation at the Chebyshev points is well behaved over the entire interval.


uniform points, n=16
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Chebyshev points, n=16
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2