An error occurred in the proof that finite subsets are subgroups if they are non-empty and closed. We chose \( H \) as the name of the finite subset of the group \( G \).

In the proof we had selected a proto-type element \( a \in H \) (whose inverse we hoped to show is in \( H \)) and then defined the mapping \( \varphi_a \) from \( H \) to \( H \) by \( \varphi_a(x) = ax \). Once we demonstrated that \( \varphi_a \) was 1-1, the Pigeon Hole principle implied that it was onto. Therefore, since \( a \in H \), \( a \) must be an image for some element \( x \) in \( H \). Consequently, we have

\[ \exists x \in H \text{ such that } \varphi_a(x) = a. \text{ Hence,} \]

\[ ax = a \]
\[ x = a^{-1}a \]
\[ x = e_G \]

(For some amazing reason I put \( ax = x \) concluding that \( a = e_G \), which is preposterous)

Please make this change in your notes. The proof goes on correctly from there to find the inverse of \( a \).