

```

/* SAS program to calculate ML parameter estimates */
/* for a multinomial likelihood function. Data are */
/* a sample of human ABO blood types. Under the */
/* Hardy-Weinberg model,  $p(\text{type A}) = a^2 + 2a \cdot o$ , */
/*  $p(\text{type B}) = b^2 + 2b \cdot o$ ,  $p(\text{type AB}) = 2a \cdot b$ , and */
/*  $p(\text{type O}) = o^2$ . The WEIGHT statement "tricks" */
/* the nonlinear regression procedure (PROC NLIN) */
/* into maximizing a multinomial likelihood with the */
/* "iteratively reweighted least squares" algorithm. */
/* Data are from Rao, C. R. 1973. Linear statistical */
/* inference and its applications, 2nd ed. Wiley. */

```

```
options nocenter ls=72;
```

```
data;
```

```
input type $ y;
```

```
cards;
```

```
a 182
```

```
b 60
```

```
ab 17
```

```
o 176
```

```
;
```

```
proc nlin nohalve sigsq=1;
```

```
parameters a=0.33 b=0.33;
```

```
n=435;
```

```
o=1-a-b;
```

```
if type='a' then do;
```

```
  p=a*a+2*a*o;
```

```
end;
```

```
if type='b' then do;
```

```
  p=b*b+2*b*o;
```

```
end;
```

```
if type='ab' then do;
```

```
  p=2*a*b;
```

```
end;
```

```
if type='o' then do;
```

```
  p=o*o;
```

```
end;
```

```
expect=n*p;
```

```
model y=expect;
```

```
_weight_=1/expect;
```

```
output out=new predicted=expect;
```

```
proc print;
```

```
run;
```

The NLIN Procedure
 Dependent Variable y
 Method: Gauss-Newton

Iter	Iterative Phase		Weighted SS
	a	b	
0	0.3300	0.3300	437.3
1	0.2476	0.0383	70.5472
2	0.2639	0.0914	1.4361
3	0.2644	0.0932	1.3758
4	0.2644	0.0932	1.3757

NOTE: Convergence criterion met.

Estimation Summary

Method	Gauss-Newton
Iterations	4
R	3.951E-6
PPC(b)	4.115E-7
RPC(b)	0.000098
Object	0.000096
Objective	1.37571
Observations Read	4
Observations Used	4
Observations Missing	0

NOTE: An intercept was not specified for this model.

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	2	435.0	217.5	316.20	0.0032
Residual	2	1.3757	0.6879		
Uncorrected Total	4	436.4			
Corrected Total	3	238.7			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
a	0.2644	0.0162	0.1947	0.3342
b	0.0932	0.0101	0.0497	0.1366

Approximate Correlation Matrix

	a	b
a	1.000000	-0.1712915
b	-0.1712915	1.000000

Obs	type	y	EXPECT
1	a	182	178.212
2	b	60	55.846
3	ab	17	21.435
4	o	176	179.508