Latin square design

Situation: two blocking variables; limited supply of experimental units or experimental capacity. The Latin square is a form of **incomplete block design**, in which each block recieves less than t treatments

ex. Manufacturing process; treatments A, B, C
Three operators (1, 2, 3): Blocking var 1
Three days of the week (M, W, F): Blocking var 2

Each operator/day combination is a block.

In the following Latin square setup, each block recieves only one treatment!

	M	W	F
1	B	A	C
2	A	C	B
3	C	B	A

Note that each treatment appears once in every row and once in every column

A $t \times t$ Latin square design has t rows and t columns. There are t treatments; each treatment appears once in every row, and once in every column. Experimental units are assigned at random; or, a Latin square is picked at random from all possible $t \times t$ Latin squares.

Model: Let Y_{ij} be the observation in the ith row, jth col, which has arisen from the kth treatment.

$$Y_{ij} = \mu + \alpha_k + \beta_i + \gamma_j + \epsilon_{ij}$$

Here μ is the overall mean, α_k is the effect of the kth treatment ($\Sigma\alpha_k = 0$), β_i is the effect of the ith row ($\Sigma\beta_i = 0$), γ_j is the effect of the jth col ($\Sigma\gamma_j = 0$), and $\epsilon_{ij} \sim$ normal$(0, \sigma^2)$.

The design is balanced, so the blocking variables and the treatments are orthogonal; formulas for the parameter estimates are:

$$\hat{\alpha}_k = \overline{y}_k - \overline{y}_{..} \qquad \widehat{\beta}_i = \overline{y}_{i.} - \overline{y}_{..} \qquad \widehat{\gamma}_j = \overline{y}_{.j} - \overline{y}_{..}$$

$$\text{SS(treatment)} = t\sum_{k=1}^{t}(\overline{y}_k - \overline{y}_{..})^2$$

$$\text{SS(row)} = t\sum_{i=1}^{t}(\overline{y}_{i.} - \overline{y}_{..})^2$$

$$\text{SS(col)} = t\sum_{j=1}^{t}\left(\overline{y}_{.j} - \overline{y}_{..}\right)^2$$

$$\text{SS(error)} = \sum_{i=1}^{t}\sum_{j=1}^{t}\left[y_{ij} - \left(\mu + \widehat{\alpha}_k + \widehat{\beta}_i + \widehat{\gamma}_j\right)\right]^2$$

$$\widehat{\sigma}^2 = \frac{\text{SS(error)}}{(t-1)(t-1)}$$

AOV table:

Source	SS	df	MS	F
row	SS(row)	$t-1$	$\frac{\text{SS(row)}}{t-1}$	$\frac{\text{MS(row)}}{\text{MS(error)}}$
col	SS(col)	$t-1$	$\frac{\text{SS(col)}}{t-1}$	$\frac{\text{MS(col)}}{\text{MS(error)}}$
trt	SS(trt)	$t-1$	$\frac{\text{SS(trt)}}{t-1}$	$\frac{\text{MS(trt)}}{\text{MS(error)}}$
error	SS(error)	$(t-1)(t-1)$	$\frac{\text{SS(error)}}{(t-1)(t-1)}$	
total	SS(total)	t^2-1		