

Three factor AOV model (factors A, B, C, with a, b, c levels, respectively)

$$Y_{ijklm} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} \\ + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijklm}$$

μ	grand mean
$\alpha_i \beta_j \gamma_k$	main effects
$(\alpha\beta)_{ij} (\alpha\gamma)_{ik} (\beta\gamma)_{jk}$	second order interactions
$(\alpha\beta\gamma)_{ijk}$	third order interaction

Effects & interactions summed on any subscript add to zero

AOV table, balanced design, n obs. per cell

source	SS	df	MS	f
A	SS(A)	$(a - 1)$	$\frac{SS(A)}{(a-1)}$	$\frac{MS(A)}{MS(error)}$
B	SS(B)	$(b - 1)$	$\frac{SS(B)}{(b-1)}$	$\frac{MS(B)}{MS(error)}$
C	SS(C)	$(c - 1)$	$\frac{SS(C)}{(c-1)}$	$\frac{MS(C)}{MS(error)}$
AB	SS(AB)	$(a - 1)(b - 1)$	$\frac{SS(AB)}{(a-1)(b-1)}$	$\frac{MS(AB)}{MS(error)}$
AC	SS(AC)	$(a - 1)(c - 1)$	$\frac{SS(AC)}{(a-1)(c-1)}$	$\frac{MS(AC)}{MS(error)}$
BC	SS(BC)	$(b - 1)(c - 1)$	$\frac{SS(BC)}{(b-1)(c-1)}$	$\frac{MS(BC)}{MS(error)}$
ABC	SS(ABC)	$(a - 1)(b - 1)(c - 1)$	$\frac{SS(ABC)}{(a-1)(b-1)(c-1)}$	$\frac{MS(ABC)}{MS(error)}$
error	SS(error)	$abc(n - 1)$	$\frac{SS(error)}{abc(n-1)}$	
total	SS(total)	$nabc - 1$		

Unbalanced design: effects are no longer orthogonal, which means that SS for effects are no longer independent of each other. There is no unique SS for a factor, and SS do not add up. The SS that can be attributed to a factor depend on which terms have already been entered.

ex. SS due to A alone + SS due to B alone do not necessarily equal SS with both A and B included.

Think of SS as *reduction of error SS as a result of entering that factor.*

Notation: $SSE(A, B)$ means the error sum of squares for the model with A and B included.

SAS: MODEL Y=A|B|C / SS1 SS2 SS3;

Type I SS is the sequential SS

source	type I SS
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A

B

C $SSE(A, B) - SSE(A, B, C)$

AB

AC $SSE(A, B, C, AB) - SSE(A, B, C, AB, AC)$

BC

ABC

Type II SS compares the particular effect with *all other terms that do not contain that particular effect*

source	type II SS
A	
B	
C	$SSE(A, B, AB) - SSE(A, B, AB, C)$
AB	
AC	$SSE(A, B, C, AB, BC) - SSE(A, B, C, AB, BC, AC)$
BC	
ABC	

Type III SS enters the particular effect *last*.

source	type III SS
A	
B	
C	$SSE(A, B, AB, AC, BC, ABC) - SSE(A, B, C, AB, AC, BC, ABC)$
AB	
AC	$SSE(A, B, C, AB, BC, ABC) - SSE(A, B, C, AB, AC, BC, ABC)$
BC	
ABC	

(Type IV SS: same as type III if there are no empty cells)