## Simple Random Sampling for Means and Totals

We have already demonstrated that

$$V(\overline{y}) = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right), \text{ and since } E(s^2) = \frac{N}{N-1}\sigma^2 \text{ (see pages 83 and 403),}$$

to get an unbiased estimator of $V(\overline{y})$ we use:

$$\widehat{V}(\overline{y}) = \frac{s^2}{n}\left(\frac{N-n}{N}\right) = \frac{s^2}{n}\left(1-\frac{n}{N}\right) = \frac{s^2}{n}\,\text{(finite population correction)}\,.$$

Note how the f.p.c. term is close to 1 if $n$ is far less than $N$; and how, in that case, it is $n$, and not $n/N$ that is important in having a small variance. In many applications, the fpc is ignored if $n < N/20$, for example. Thus for estimation of $\mu$ we have:

$$\widehat{\mu} = \overline{y} = \frac{\sum_{i=1}^{n} y_i}{n} \text{ and } \widehat{V}(\overline{y}) = \frac{s^2}{n}\left(\frac{N-n}{N}\right),$$

and for estimation of $\tau$ we have:

$$\widehat{\tau} = N\overline{y} \text{ and } \widehat{V}(\widehat{\tau}) = \widehat{V}(N\overline{y}) = N^2\widehat{V}(\overline{y}) = N^2\frac{s^2}{n}\left(\frac{N-n}{N}\right).$$

## Examples

## Sample Size selection for Means and Totals

If we know the population size $N$, the desired bound $B$, and have an idea of the variance $\sigma^2$, we can derive a required sample size. To do this, we use the expression for the bound, and solve for the sample size $n$. For the mean $\mu$, we have:

$$2\sqrt{V(\overline{y})} = B \quad \text{or equivalently} \quad 2\sqrt{\frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)} = B \text{ , solving for } n \text{ yields:}$$

$$n = \frac{N\sigma^2}{(N-1)(B^2/4) + \sigma^2}.$$

Using the same approach for estimation of the the total $\tau$ gives:

$$n = \frac{N\sigma^2}{(N-1)(B^2/4N^2) + \sigma^2}.$$

When calculating sample size, how to you choose a value for $\sigma^2$? Some options are i) use estimates from a pilot study, ii) use estimates from the literature, or iii) use an approximation of $\sigma \approx$ range/4 .

## Examples