Simple Random Sampling for Means and Totals

We have already demonstrated that

\[ V(\bar{y}) = \frac{\sigma^2}{n} \left( \frac{N - n}{N - 1} \right) \],

and since \( E(s^2) = \frac{N}{N - 1} \sigma^2 \) (see pages 83 and 403),

to get an unbiased estimator of \( V(\bar{y}) \) we use:

\[ \hat{V}(\bar{y}) = \frac{s^2}{n} \left( \frac{N - n}{N} \right) = \frac{s^2}{n} \left( 1 - \frac{n}{N} \right) = \frac{s^2}{n} \text{ (finite population correction)} \].

Note how the f.p.c. term is close to 1 if \( n \) is far less than \( N \); and how, in that case, it is \( n \), and not \( n/N \) that is important in having a small variance. In many applications, the fpc is ignored if \( n < N/20 \), for example. Thus for estimation of \( \mu \) we have:

\[ \hat{\mu} = \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \quad \text{and} \quad \hat{V}(\bar{y}) = \frac{s^2}{n} \left( \frac{N - n}{N} \right), \]

and for estimation of \( \tau \) we have:

\[ \hat{\tau} = N\bar{y} \quad \text{and} \quad \hat{V}(\hat{\tau}) = \hat{V}(N\bar{y}) = N^2 \hat{V}(\bar{y}) = N^2 \frac{s^2}{n} \left( \frac{N - n}{N} \right). \]

Examples
Sample Size selection for Means and Totals

If we know the population size $N$, the desired bound $B$, and have an idea of the variance $\sigma^2$, we can derive a required sample size. To do this, we use the expression for the bound, and solve for the sample size $n$. For the mean $\mu$, we have:

$$2\sqrt{V(\bar{y})} = B \quad \text{or equivalently} \quad 2\sqrt{\frac{\sigma^2}{n} \left(\frac{N - n}{N - 1}\right)} = B,$$

solving for $n$ yields:

$$n = \frac{N\sigma^2}{(N - 1)(B^2/4) + \sigma^2}.$$

Using the same approach for estimation of the total $\tau$ gives:

$$n = \frac{N\sigma^2}{(N - 1)(B^2/4N^2) + \sigma^2}.$$ 

When calculating sample size, how do you choose a value for $\sigma^2$? Some options are i) use estimates from a pilot study, ii) use estimates from the literature, or iii) use an approximation of $\sigma \approx \text{range}/4$.

Examples