Fixed effects Completely Randomized Design model

In this model,

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij},$$

the t values of τ_i, $\tau_1, \tau_2, ..., \tau_t$, are the only effects of interest, and ε_{ij} is the only random term, ε_{ij} has a normal distribution with mean 0 and variance σ_e^2 ($\varepsilon_{ij} \sim N(0, \sigma_e^2)$).

Random effects Completely Randomized Design model

In this model,

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij},$$

The τ_i terms are now random, $\tau_i \sim N(0\ ,\ \sigma_t^2\ )$, and the τ_i and ε_{ij} are independent.

Now the τ_i 's are viewed as a random sample from a population of τ_i 's, and the ANOVA H_0 is H_0: $\sigma_\tau^2 = 0$ vs. $H_a : \sigma_\tau^2 > 0$.

How do you know if an effect is fixed or random?

1. How were the levels for the factor in question chosen?
2. Is it desired to generalize the results to levels that weren't used?

Implications for random effects

1. The denominator of the F statistic may change.
2. Generally you are not interested in doing multiple comparisons. There may be interest in estimating σ_τ^2 , σ_e^2 , etc.
3. Estimating σ_t^2 may still be of interest even if we reject $H_0 : \sigma_{\tau\beta}^2 = 0$

Random effects model for Randomized Block Design

The model is:

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij},$$

Where $\tau_i \sim N(0, \sigma_\tau^2)$, $\beta_j \sim N(0, \sigma_\beta^2)$, $\varepsilon_{ij} \sim N(0, \sigma_e^2)$, and τ_i , β_j , and ε_{ij} are mutually independent. For the RB design (no replication) we assume that $\sigma_{\tau\beta}^2 = 0$.

Generalized Randomized Block Design with random effects

1. Test $H_0 : \sigma^2_{\tau\beta} = 0$ using F = MSAB/MSE

2. If the P value for $H_0 : \sigma^2_{\tau\beta} = 0$ is large ($>$ .15 or $>$ .25), you may choose to pool terms, creating MSE(pooled) = (SSAB +SSE)/(dfAB + dfE), also denoted by MSRES.

Mixed Effects Models

A **mixed effect model** includes both fixed and random effects. The lotions for allergies experiment is an example of a mixed model.

In the two factor random effect model, E(MSA) = $\sigma^2_e + n\ \sigma^2_{\tau\beta} + bn\ \sigma^2_\tau$, and E(MSAB) = $\sigma^2_e + n\ \sigma^2_{\tau\beta}$.

When $H_0 : \sigma^2_\tau = 0$ is true, then E(MSA) = E(MSAB) and the sample F value is close to 1.

However, E(MSE) = σ^2_e so even if the null hypothesis for factor A is true ($\sigma^2_\tau = 0$) the sample F value will tend to be greater than 1 if we use F = MSA/MSE .

Instead we use F = MSA/MSAB.