1 Review problem set 5 solutions

Here are solutions to review problem set 5:

9.3 a) Linear combinations 1, 2, and 4 have coefficients that satisfy $\sum a_i = 0$, so they are contrasts.

b) Contrasts 2 and 4 are orthogonal, since the coefficients for l_2 are 1 1 0 -2 and for l_4 they are 1 1 -3 1, so when we multiply corresponding coefficients we get (1)(1) +(1)(1) +(0)(-3) +(-2)(1) = 0. Since the sum is equal to zero, and (presumably) their sample sizes are equal, they are orthogonal.

c) Yes those two contrasts are testing the same hypothesis, because $l_1 = 3l_2$, so if one of them is 0 then they both are equal to 0.

9.14 a) LSD $= t_{\alpha/2,df}\sqrt{MSE\,(\frac{1}{n_i}+\frac{1}{n_j})} = t_{.025,45}\sqrt{.982(\frac{1}{10}+\frac{1}{10})} \approx (2.021)\sqrt{.982(\frac{1}{10}+\frac{1}{10})} =$ .896. See the computer printout for results of mean comparisons.

b) W $= q_{t,df,\alpha}\sqrt{MSE\,(\frac{1}{n})} = q_{5,45,.05}\sqrt{.982(\frac{1}{10})} = (4.04)\sqrt{.982(\frac{1}{10})} = 1.27$. See the computer printout for results of mean comparisons.

15.6 a) The model is $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$. To estimate the effects, just use the sample means. For example, $\alpha_{classical} = \mu_{classical\cdot} - \mu_{nomusic\cdot}$, so our estimate is $\widehat{\alpha}_{classical} = \overline{y}_{classical\cdot} - \overline{y}_{nomusic\cdot} = 23.0 - 20.86 = 2.14$.

b) Yes, $F = 6.54, P = .012$ so we reject $H_0 : \alpha_{classical} = \alpha_{nomusic} = \alpha_{hardrock} = 0$.

c) From the plot of the data, the additivity assumption seems reasonable as there is little evidence of lack of parallelism.

d) $RE = \frac{(b-1)MSB+b(t-1)MSE}{(bt-1)MSE} = [6(24.9) + 14(2.37)]/[20(2.37)] = 3.85$. Yes, the blocks were effective. We would need approximately 3.85 times as many observations per treatment group to obtain the same precision if we had conducted a completely randomized design instead of this randomized block design.

15.7) Yes, the residuals appear normal and the residual-by-predicted plot shows a roughly horizontal band of data points.

15.10 a) The model is $y_{ijk} = \mu + \tau_k + \beta_i + \gamma_j + \varepsilon_{ijk}$.

b) See the printout, as above the parameter estimates can be obtained from the means.

c) From the ANOVA, $F = 8.22$, $p = .015$, so reject H_0 at $\alpha = .05$.

d) From the Tukey results, gasoline blend C has lower MPG than blends B or D.

e) $RE = \frac{MSR+MSC+(t-1)MSE}{(t+1)MSE} = [2.78 + 251.8 + 3(4.31)]/[5(4.31)] = 12.4$. We would need approximately 12.4 times as many observations per treatment group to obtain the same precision if we had conducted a completely randomized design instead of this Latin Square design. Cars were very effective in reducing variability, but drivers were not.

f) If a future study was planned that was similar to the present study, perhaps a RCB design using only cars would be a better choice.

14.23 a) The profile plot shows clear evidence of interaction.

b) The model is: $y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk}$.

c) The ANOVA results for these data can be obtained from the SAS code and are displayed in the next problem.

14.24 a) For interaction, $F = 8.00, P < .0001$ so we reject $H_0 : \alpha\beta_{11} = \alpha\beta_{12} = ... = \alpha\beta_{43} = 0$. For CA, $F = 10.82, P = .0004$ so we reject $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$. For PH, $F = 21.94, P < .0001$ so we reject $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$.

b) Referring to the profile plot in 15.24a, we see that there is an interaction between Calcium and pH so that, for example, the comparison of Calcium levels depends upon which pH is being applied.

14.25 a) W $= q_{t,df,\alpha}\sqrt{MSE\,(\frac{1}{n})} = q_{3,24,.05}\sqrt{.0678\,(\frac{1}{12})} = 3.53\sqrt{.0678\,(\frac{1}{12})} = .265$. We should not continue with the mean comparisons because of the interaction.

b) If we are comparing the mean response among calcium levels separately by pH, then we should use W $= q_{t,df,\alpha}\sqrt{MSE\,(\frac{1}{n})} = q_{12,24,.05}\sqrt{.0678\,(\frac{1}{3})} = 5.10\sqrt{.0678\,(\frac{1}{3})} = .767$,which leads to different conclusions for different pH values. We are using $t = 12$ because we are exploring pairwise combinations that are a subset of all interaction pairwise combinations, thus $t = 3 * 4 = 12$.

For the cuckoo data problem, the contrast is $L = (\mu_{TP} + \mu_{MP})/2 - (\mu_{HS} + \mu_{PW} + \mu_R + \mu_W)/4$. Our estimate of L is $\widehat{L} = (\overline{y}_{TP} + \overline{y}_{MP})/2 - (\overline{y}_{HS} + \overline{y}_{PW} + \overline{y}_R + \overline{y}_W)/4 = (22.28+23.09)/2-(23.17+22.90+22.54+21.13)/4 = .25$. The standard error of the contrast is $\widehat{s.e.}(\widehat{L}) = \sqrt{MSE\sum \frac{a_i^2}{n_i}} = \sqrt{(.904)(\frac{1}{15})[4(-\frac{1}{4})^2 + 2(\frac{1}{2})^2]} = .213$. Thus the t statistic is $t = .25/.213 = 1.18$, which we compare to $t_{84,.025} \simeq 2$, so we fail to reject $H_0 : L = 0$. For Scheffe's method, we compare $\widehat{L}$ to $S = \sqrt{MSE\sum \frac{a_i^2}{n_i}}\sqrt{(t-1)F_{.05,t-1,df(MSE)}} = .213\sqrt{5F_{.05,5,84}} = .213\sqrt{5(2.37)} = .73$. Since $\widehat{L} < |S|$, we fail to reject $H_0 : L = 0$.