Treatment structure equivalence shown with contrasts

As mentioned in class, a factorial treatment structure with a levels of factor A and b levels of factor B (a two way ANOVA) can equivalently be seen as a completely randomized structure (one way ANOVA) with ab levels. For example, take our hotdog example with just two levels of meat ($Beef$ and $Chicken$) and two levels of salt (1 and 2). There are four combinations of these two factors, so we could view it as a one-way ANOVA with four levels, with 3 df between groups. The four means can be designated as:

$$\mu_{B1}, \mu_{B2}, \mu_{C1}, \text{ and } \mu_{C2}$$

We have discussed how the ANOVA H_0 of equality of the four means is equivalent to specifying three contrasts that we test simultaneously. Here are three possible contrasts:

$$\begin{aligned}
&(\mu_{B1} + \mu_{B2}) - (\mu_{C1} + \mu_{C2}) = \mu_{B1} + \mu_{B2} - \mu_{C1} - \mu_{C2} \\
&(\mu_{B1} + \mu_{C1}) - (\mu_{B2} + \mu_{C2}) = \mu_{B1} - \mu_{B2} + \mu_{C1} - \mu_{C2} \text{ and} \\
&(\mu_{B1} - \mu_{B2}) - (\mu_{C1} - \mu_{C2}) = \mu_{B1} - \mu_{B2} - \mu_{C1} + \mu_{C2}.
\end{aligned}$$

If these three contrasts are examined, it can be seen that they represent the effects of the meat factor, the salt factor, and their interaction respectively. This is an orthogonal set of contrasts, which means that each pair of contrasts are orthogonal. With orthogonal sets of contrasts, if we have as many contrasts as degrees of freedom for a factor, then the sums of squares of the contrasts equals the sum of squares for the factor. The accompanying SAS code illustrates this concept.

Notice in this example that if we have a factor with only 1 degree of freedom, we can use a single contrast and its sum of squares equals the factor's sum of squares. In these cases we often say that the contrast represents the effect of that factor. For example the first contrast above is the effect of the meat factor.