Randomized Complete Block Design Topics; Latin Square Designs

Some issues that arise with RCB designs:

The additivity assumption

The text discusses Tukey's one-degree of freedom test for nonadditivity. This test detects a particular type of nonadditivity (multiplicative) but there are also other approaches. The SAS code for the lecture implements a more general test presented in Oehlert (2000) that also suggests a power transformation for use when nonadditivity exists.

Related RCB designs

If we have multiple experimental units per treatment per block, then we can conduct traditional tests for interaction, this design is sometimes called a generalized randomized complete block design. The text also discusses block designs with subsampling.

The usual assumptions, did the blocking work?

The text discusses the use of residual plots as discussed in previous chapters to check the usual assumptions on the errors. One way to assess the use of blocking is through the concept of relative efficiency. The relative efficiency of the RCB design to a completely randomized design is:

$$\text{corrected } RE = \frac{(f_{rcb}+1)(f_{cr}+3)}{(f_{rcb}+3)(f_{cr}+1)} \frac{s^2_{cr}}{s^2_{rcb}}$$

where s^2_{rcb} is the mean squared error from the RCB analysis, f_{rcb} is the error degrees of freedom for the RCB design, f_{cr} is the error degrees of freedom if the data had come from a CR design, and s^2_{cr} is estimated by:

$$s^2_{cr} = \frac{SS \text{ Blocks } + r(t-1)MSE}{rt-1}$$

The authors also mention that the ratio MSB/MSE from the RCB analysis can be useful as a quick guide to the usefulness of blocking.

Random blocks

In many applications it is reasonable to assume that the blocks are a random effect. In these cases the ANOVA test for H_0 for the fixed effect does not change, and the standard error of the differences between means does not change, but the standard error of individual treatment means does change.

Latin Square Designs

Latin Square designs are used to achieve error reduction when there are two blocking variables, and both of them and the treatment factor have the same number of levels. As mentioned in the text, the restriction to have the same number of levels makes Latin square designs less useful for a small number of treatment levels, and fairly costly for a large number of treatment levels. The layout of a typical design is shown, and the procedure to randomize a Latin square design is illustrated nicely in the text. The model for a Latin square design is:

$$y_{ij} = \mu + \rho_i + \gamma_j + \tau_k + e_{ij}$$

with all three terms having t levels and the usual assumptions on the error terms e_{ij}. Many of the same topics discussed for RCB designs are discussed, such as a test for nonadditivity, and calculation of row or column relative efficiencies. The idea of having multiple Latin squares is discussed to help with error degrees of freedom when the number of treatment levels is small.

References

Oehlert, G.W. 2000. A First Course in Design and Analysis of Experiments. New York: W.H. Freeman and Company.