1 Chapter 5: Further topics in Random effects models

1.1 The Intraclass Correlation Coefficient

One measure of interest in a random effects model is the ratio of the variance of the random effect to the total variance of the response. This quantity is called the intraclass correlation coefficient:

$$\rho_I = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}$$

As mentioned in the text, ρ_I is a measure of similarity of the observations within the same treatment group level. The text shows how to calculate estimates of the coefficient using method of moments estimators, but we will use a maximum-likelihood-based estimator, as shown in the code.

1.2 Power and sample size for the random effects model

For the completely randomized design with random effects and one-way treatment structure, when the null hypothesis is true, the F statistic has an F distribution with $t-1$ and $t(r-1)$ degrees of freedom. When the null hypothesis is false, the distribution of the F statistic is a multiple of a central F distribution with $t-1$ and $t(r-1)$ degrees of freedom, where the multiplicative constant is equal to:

$$\lambda^2 = 1 + \frac{r\sigma_a^2}{\sigma_e^2},$$

so that we can calculate the power by

$$P\left[F > \frac{1}{\lambda^2} F_{\alpha,\nu_1,\nu_2}\right],$$

where λ^2 is the value of the multiplicative constant under H_a and F_{α,ν_1,ν_2} is a tabled F value for $\alpha =$ Type I error, and degrees of freedom ν_1 and ν_2. Power can then be calculated by specifying a value for the ratio σ_a^2/σ_e^2, or via the way presented in the text where a percentage P increase in σ_y caused by σ_a^2 is specified.

1.3 Random subsamples

Sometimes subsamples are collected from an experimental unit and measured separately. We need to use a model that reflects the distinction between variation between experimental units and between subsamples within an experimental unit. The model is:

$$y_{ij} = \mu + \tau_i + e_{ij} + d_{ijk},$$

where d_{ijk} is the random effect of the k^{th} subsample within the j^{th} experimental unit in the i^{th} treatment, and the other terms are as defined in previous models. The text shows the expected value of the mean squares of these terms, which are used to determine the form of the F test. The use of expected mean squares to determine how to form the appropriate F statistics to test hypotheses of interest will become increasingly important.

1.4 Random subsamples with unequal sample sizes

At the end of the chapter, the random subsample model is considered again, but now with unequal replicate and subsample numbers. The table of expected mean squares is shown on page 167:

Source of variation	Expected mean squares
Treatments	$\sigma_d^2 + c_1\sigma_e^2 + c_2\sigma_a^2$
Error	$\sigma_d^2 + c_3\sigma_e^2$
Sampling	σ_d^2

Unless $c_1 = c_3$, the ratio MST/MSE does not have the proper behavior under the null hypothesis. As shown in the text, in these cases we can construct a linear function of mean squares:

$$M = a_1 MSE + a_2 MST$$

that will have expected value $\sigma_d^2 + c_1\sigma_e^2$ so that an appropriate F ratio can be formed. For these linear functions of mean squares:

$$M = a_1 MS_1 + a_2 MS_2 + \cdots + a_k MS_k$$

it was shown by Satterthwaite (1946) that the distribution of this linear function M can be approximated by a chi-square distribution with degrees of freedom equal to:

$$\nu = \frac{M^2}{\sum_{i=1}^{k} \frac{(a_i MS_i)^2}{\nu_i}}$$

2 Chapter 6: Factorial treatment designs

Several factors can be examined simultaneously in a **factorial treatment design,** which includes all possible combinations of the levels of several factors. Three effects of interest in a factorial experiment are **simple effects**, **main effects**, and **interaction effects**.

Visualizing effects in profile plots

The cell means model

The two-factor analysis: $(y_{ijk} - \overline{y}_{\bullet\bullet\bullet}) = (\overline{y}_{ij\bullet} - \overline{y}_{\bullet\bullet\bullet}) + (y_{ijk} - \overline{y}_{ij\bullet})$ which leads to SSTotal= SSTreatment+SSError, and for balanced designs we have $SS\text{Treatment} = SSA + SSB + SSAB$ giving us

$$SS\text{Total} = SSA + SSB + SSAB + SS\text{Error}$$

As noted in the text, the presence of interaction affects the interpretation of main effect results.