1 Factorial Experiments, Treatment Structure, and Analysis

An experiment with more than one factor is a factorial experiment if all factor-level combinations are used. Two advantages of this approach are i) the ability to detect interactions between treatments, and ii) more efficiency (for all tests) than separate one-at-a-time experiments. The nonstarter bacteria study from our text is an example of a factorial experiment. Even if the data do not come from an experiment, but the grouping factor is arranged into all combinations of some other factors, then we can still say that the groups have factorial treatment structure. As an example, suppose that we measure blood pressure on six groups of people: Male children, Female children, Male young adults, Female young adults, Male middle aged adults, and Female middle ages adults. The factors of gender and age are not randomly applied, but we can talk about the six groups having factorial treatment structure in these two factors. Initially we will consider models for data where only two factors are used, and also we will at first only consider balanced data, in which all treatment combinations have the same number, n , of replicates.

1.1 Models for data with factorial treatment structure

With data that have factorial treatment structure, we can consider either of two models. The first model we have used already:

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij},$$

where now $i = 1, ..., g$, and the g groups consist of all treatment combinations of the factors. Alternatively, we can use the model

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk},$$

where $i = 1, ..., a$, and $j = 1, ..., b$, index the two treatment factors and $k = 1, ..., n$ indexes the replicates within a treatment combination. Note that the new model has many more parameters than the g treatment combinations. As discussed in the text, we commonly use zero-sum restrictions to enable us to obtain unique estimators of these parameters. The new (factorial) model is quite useful for describing the separate effects of the individual factors, particularly if there is no interaction or easily described interaction. For very complicated patterns of interaction it can be preferable to revert to the earlier model with g groups.

Important topics in the analysis of data with factorial treatment structure include how to perform the ANOVA calculations, understanding the meaning of the parameters α_i, β_j, and $\alpha\beta_{ij}$, how to visualize and further interpret interactions.