An Approximate F test example

In the text, Exercise 6.7 has data from a balanced three-factor experiment. Although it is used with fixed effects there, we will assume the effects are random for this example. To correspond to our earlier notation, we will use factor A = soil, factor B = salinity, and factor C = water for these data. The SAS file using data from Exercise 6.7 gives output that shows $MS_C = 13.14$, $MS_{AC} = .759$, $MS_{BC} = 5.15$, and $MS_{ABC} = .206$, with degrees of freedom of $2, 4, 4$, and 8 respectively. The linear combination of mean squares equals:

$$M = MS_{AC} + MS_{BC} - MS_{ABC} = .759 + 5.15 - .206 = 5.7,$$

and its degrees of freedom is:

$$\nu = \frac{M^2}{\sum \frac{(a_i MS_i)^2}{\nu_i}} = \frac{A}{B}$$

where

$$A = 5.7^2$$

and

$$B = \sum \frac{(a_i MS_i)^2}{\nu_i} = \frac{(1 * 0.759)^2}{4} + \frac{(1 * 5.15)^2}{4} + \frac{(-1 * 0.206)^2}{8} = 6.78.$$

So our F statistic is

$$F = \frac{MS_C}{MS_{AC} + MS_{BC} - MS_{ABC}} = \frac{13.14}{5.7} = 2.3$$

on degrees of freedom of 2 and $\nu = A/B = 5.7^2/6.78 = 32.5/6.78 = 4.8$.

The output from the SAS program that uses a RANDOM /TEST statement includes a table of expected mean squares and results for the denominator mean square, the degrees of freedom for the denominator mean square, and the F value that match those shown above.