Factorial Treatment Designs: Random and Mixed Models

In Chapter 6 we considered factorial treatment designs for fixed effect models. In this chapter we expand the use of factorial treatment designs to cases where each factor is random (**Random Effects Factorial**) and where some factors are fixed and others are random (**Mixed Effects Factorial**). There are nice examples of each case in the beginning sections of the chapter. For the two factor random effects case, the model is:

$$y_{ijk} = \mu + a_i + b_j + (ab)_{ij} + e_{ijk},$$

where every random term is assumed independent of the others and normally distributed with mean 0 and its own variance component. The issues that arise with these designs are similar to what we have seen in previous chapters, including the need to use expected mean squares to construct F tests, which sometimes have to be approximate F tests. In some instances there are alternate ways to construct approximate F tests, and if possible a linear combination of mean squares should be chosen with all positive coefficients, as shown in an example. For the two factor mixed effects case, the model is:

$$y_{ijk} = \mu + \alpha_i + b_j + (ab)_{ij} + e_{ijk},$$

where the b_j , $(ab)_{ij}$, and e_{ijk} terms are assumed independent and normally distributed with 0 means and their own variance components. Here too, the analyses must take account of expected mean squares. Method of moments estimators of variance components are presented, and expressions are given for standard errors of means and differences of means - a topic we will examine in more detail in a future chapter. The text also mentions an alternate formulation of the mixed effects model which restricts the mixed interaction terms to sum to zero over the fixed effects, hence it is called the restricted model. The text makes some arguments in favor of the original model (the unrestricted model), which is also the model assumed by SAS in its calculations.