Treatment Designs with Nested or Nested and Crossed Factors

Designs with nested factors, in which the levels of one factor (B) are not identical across all levels of another factor (A) occur commonly in many types of research. The model for a nested design with three random factors, A, B within A, and C within B, is:

$$y_{ijk} = \mu + a_i + b_{j(i)} + c_{k(ij)},$$

where every random term is assumed independent of the others and normally distributed with mean 0 and its own variance component. It is not uncommon for the A factor to be a fixed effect, and the text gives an example of a situation in which both the A and B factors are fixed effects. As with other designs we use the information from the expected mean squares for each effect to determine which F ratios to use to test hypotheses. Method of moments estimators of variance components are presented, and expressions are given for standard errors of means. The use of staggered nested designs to yield more balanced degrees of freedom across effects is discussed.

Some experimental designs use both crossed and nested effects, often called a nested factorial design. One example would be if we had a fixed factor A, a random factor B crossed with A, a random factor C nested within B, with replicate samples within the nested factor. An example of this situation is given in the text, and would have a model of:

$$y_{ijkl} = \mu + \alpha_i + b_j + c_{k(j)} + (ab)_{ij} + (ac)_{ik(j)} + e_{ijkl},$$

where the b_j , $c_{k(j)}$, $(ab)_{ij}$, $(ac)_{ik(j)}$, and e_{ijkl} terms are assumed independent and normally distributed with 0 means and their own variance components. Here too, the analyses must take account of expected mean squares. See the SAS code for examples of analyzing data from these designs.