

Randomized Complete Block Designs

As we have seen in previous examples, in many cases there is variation in a response due to a factor other than the experimental treatment. Taste test examples are always good illustrations of this idea. In a randomized complete block design, we attempt to control this extraneous variation by applying each level of the treatment to homogeneous units called blocks. Blocks could be people, contiguous areas in a field, etc. The RCB design uses a restricted randomization, in which the set of treatment levels is separately randomized in each block so that each block receives each treatment level once. The linear model for an RCB is:

$$y_{ij} = \mu + \tau_i + \rho_j + e_{ij},$$

where the treatment and block effects are assumed to be additive, meaning that they do not interact. Some issues that arise with RCB designs:

The additivity assumption

The text discusses Tukey's one-degree of freedom test for nonadditivity. This test detects a particular type of nonadditivity (multiplicative) but there are also other approaches. The SAS code for the lecture implements a more general test presented in Oehlert (2000) that also suggests a power transformation for use when nonadditivity exists.

Related RCB designs

If we have multiple experimental units per treatment per block, then we can conduct traditional tests for interaction, this design is sometimes called a generalized randomized complete block design. The text also discusses block designs with subsampling.

The usual assumptions, did the blocking work?

The text discusses the use of residual plots as discussed in previous chapters to check the usual assumptions on the errors. One way to assess the use of blocking is through the concept of relative efficiency. The relative efficiency of the RCB design to a completely randomized design is:

$$\text{corrected } RE = \frac{(f_{rcb} + 1)(f_{cr} + 3)}{(f_{rcb} + 3)(f_{cr} + 1)} \frac{s_{cr}^2}{s_{rcb}^2}$$

where s_{rcb}^2 is the mean squared error from the RCB analysis, f_{rcb} is the error degrees of freedom for the RCB design, f_{cr} is the error degrees of freedom if the data had come from a CR design, and s_{cr}^2 is estimated by:

$$s_{cr}^2 = \frac{SS \text{ Blocks} + r(t-1)MSE}{rt - 1}$$

The authors also mention that the ratio MSB/MSE from the RCB analysis can be useful as a quick guide to the usefulness of blocking.

Random blocks

In many applications it is reasonable to assume that the blocks are a random effect. In these cases the ANOVA test for H_0 for the fixed effect does not change, and the standard error of the differences between means does not change, but the standard error of individual treatment means does change.

References

Oehlert, G.W. 2000. A First Course in Design and Analysis of Experiments. New York: W.H. Freeman and Company.