1 Incomplete Block Designs

Incomplete block designs are needed when we cannot fit all levels of a treatment into our blocks. The book has a good example where the treatment levels are temperatures for tomato seed germination.

1.1 Balanced Incomplete Block Designs (BIBD)

In this design, each block has k units, which is not equal to the total (t) number of treatment levels. The 'balanced' condition refers to the fact that in this design all pairs of treatment levels occur together in the same number of blocks, so that the variance of estimated treatment differences is the same for all pairs of treatments. If each treatment is applied to r units, and b blocks are used, the total number of units N satisfies $N = kb = rt$. As discussed in the text, the number of times that two treatments occur together is $\lambda = r(k-1)/(t-1)$, so if λ is not a whole number then a BIBD does not exist for that combination of k, b, r, and t. A BIBD always exists, however, as long as $k < t$. The text gives a nice description of a three-step process of randomization in a BIBD. The usual analysis of data from a BIBD, called the intrablock analysis, consists of obtaining the sums of squares for treatments after first adjusting for blocks - this is just the Type III SS approach that we have seen before for analyzing unbalanced factorial experiments. The text introduces the concept of recovery of interblock information, which is particularly useful when blocking has not been effective. The text also discusses the efficiency factor for BIB designs relative to RCB designs.

1.2 Related topics

When we want to block on two factors but do not have blocks large enough for a Latin square, we can use Youden squares, which can be obtained by omitting rows from a Latin square design. The usual analysis of a Youden square is the same kind of intrablock analysis as mentioned for BIBD's. If it is not possible to construct a BIBD in a particular situation, it may be possible to construct a slightly more complicated design called a partially balanced incomplete block design (PBIBD). In this design, not all pairs of treatments occur together in the same number of blocks. We use the concept of associate classes to describe this more complicated arrangement of treatments into the blocks. An associate class is a set of treatment pairs where each pair from the set occur together the same number of times, λ_i . The same kind of intrablock analysis used for BIBD's and Youden squares is also usually used for these designs.