Confounded Block Designs

In previous chapters we encountered incomplete block designs for a single factor, and one approach (BIBD) dealt with this problem by retaining the same amount of information about each pair of treatment levels. In Chapters 15 and 18 we show that for $2^k$ factorial experiments ($k$ factors each at 2 levels) we can deliberately choose to sacrifice information on certain effects to enable us to test other effects with smaller blocks or experiments.

Before introducing the concept of confounding, we need to familiarize ourselves with some ideas and notation about $2^k$ factorial experiments. As explained in the text, in a $2^k$ experiment each effect has 1 degree of freedom and can be specified by a single contrast among the $2^k$ treatment levels. Also, there is a widely-recognized notation used where each factor can be seen as 'absent' or 'present' and we can specify a treatment combination by indicating which factors are at the 'present' level. Thus for a $2^3$ experiment, the treatment combination $a$ is where the A factor is present and the B and C factors are absent. The combination $abc$ indicates that all three factors are present and the combination (1) indicates that all three factors are absent. Other conventions are that a single factor can be coded as '-' or '+' for the absent and present levels, respectively. Sometimes $'0'$ and $'1'$ are used instead of '-' and '+'. These conventions are illustrated for $2^3$ factorial experiment in Table 15.1.

If we cannot fit all $2^k$ treatment combinations into blocks, we can pick a half fraction and use $2^{k-1}$ combinations per block (this can be extended to smaller fractions also). We do this by choosing an effect to confound (typically the highest order interaction) and using a table like Table 15.1, we put all combinations with a '+' sign for the confounded effect into a block and all combinations with a '-' sign into a different block. We then get replicates of these incomplete blocks, as illustrated by our burger example outlined in Example 15.1. The burger example is a completely confounded $2^3$ block design with blocks confounded with the $ABC$ interaction. A slightly different approach is to partially confound several effects by confounding a different effect in each pair of blocks, as illustrated in Table 15.7 with its ANOVA shown in Listing 15.1. There is SAS and R code for the burger example and a partially confounded design example on the website.

We can achieve smaller block sizes in confounded designs by confounding two effects simultaneously as shown in Table 15.4. There a $2^{4-2}$ confounded block design results by choosing treatment levels for a block that have the same sign for both the $ABC$ effect and the $BCD$ effects, and we thus have a block size of 4. As noted in the text, whenever we confound two effects, another effect is also confounded called the generalized interaction of the two effects. A simple multiplication rule for finding generalized interactions is shown in the text, and a general method based on modular arithmetic is also shown in the chapter.