Split-Plot Designs

The basic split plot design with two factors is characterized by having a different size of experimental unit for each of the two treatment factors. Also, there are two distinct randomizations, one for each size of unit. When examining data and deciding how to analyze it, it is essential to know how the randomization for the experiment was performed - otherwise you cannot tell whether the design is completely randomized factorial, randomized complete block, split plot, or something else. The whole plot structure for the two factor split plot can have different designs, such as completely randomized, randomized complete block, or latin square, as shown in the text. The model for a split plot design with completely randomized whole plot structure is:

$$y_{ijk} = \mu + \alpha_i + \eta_{k(i)} + \beta_j + (\alpha\beta)_{ij} + e_{k(ij)},$$

and the model for a split plot design with randomized complete block whole plot structure is:

$$y_{ijkl} = \mu + \alpha_i + \gamma_k + \eta_{l(ik)} + \beta_j + (\alpha\beta)_{ij} + e_{l(ijk)}.$$

Referring to the expected mean squares for the design shows that the whole plot test is conducted using the whole plot error, and the split plot tests are conducted using the split plot error. Although the output from an ordinary ANOVA program can be adjusted to obtain correct F tests for both factors (or in SAS the RANDOM statement with the TEST option works), the standard errors of mean differences may require Satterthwaite adjusted degrees of freedom for simple effect comparisons. Standard fixed-effect programs (like Proc GLM in SAS or the lm function in R) cannot correctly obtain these degrees of freedom, so it is better to use mixed model programs to analyze data from these designs. Comparing the split-plot design to a randomized complete block factorial design, we find that the whole plot test is generally less efficient while the split plot tests are usually more efficient.