1 Chapter 3: Completely Randomized Design (CRD) Part II

[**The decomposition of research data into sums of squares and degrees of freedom, along with the determination of how to form F ratios may not always be the most efficient way to analyze a set of data. However, it is probably the best way to think about how to analyze data.**]

As is shown at the end of Chapter 3 (section 3.7), $E(MSE) = \sigma^2$ and $E(MS_{Trt}) = \sigma^2 + \theta_t^2$ where

$$\theta_t^2 = \frac{\sum n_i \alpha_i^2}{(g-1)},$$

so when the null hypothesis is true, $E(MSE) = E(MS_{Trt})$ and the F statistic value should be close to 1.

1.1 A general principle for GLM tests, standard errors and conf. intervals

As noted in the text, a general way to conduct tests for linear models (and even more generally) is to obtain full and reduced models as illustrated above. An F test is obtained by measuring the reduction in sum of squared error in the two models compared to the sum of squared error for the full model, with both terms divided by their degrees of freedom to create mean squares:

$$F = \frac{(SSE_r - SSE_f)/(g-1)}{SSE_f\,/(N-g)}$$

Note that the text uses SSR_0 and SSR_A instead of SSE_r and SSE_f , respectively. Under the standard ANOVA assumptions, this F statistic follows an F distribution with $g-1$ and $N-g$ degrees of freedom. Standard errors for group means are obtained by dividing a variance estimate by the group sample size, so a standard error for the mean $\overline{y}_i$ will be:

$$s_{\overline{y}_i} = \sqrt{\frac{\widehat{\sigma}^2}{n_i}},$$

where n_i is the group sample size and

$$\widehat{\sigma}^2 = \frac{\sum_{i=1}^{g}\sum_{j=1}^{n_i}(y_{ij} - \overline{y}_{i.})^2}{N-g} = \frac{SSE}{N-g}$$

is the mean squared error for the data.

2 References

Casella, G. 2008. Statistical Design, Springer.

Kuehl, R.O. 2000. Statistical Principles of Research Design and Analysis (second edition), Duxbury Press.

Kirk, R.E. 1994. Experimental Design: Procedures for Behavioral Sciences (third edition), Wadsworth Publishing.

Oehlert, G.W. 2000. A First Course in Design and Analysis of Experiments, W.H. Freeman. (now out of print, but available for free under Creative Commons license at Prof. Oehlert's website at the University of Minnesota)