Fixed versus Random effects models

So far all of the treatment effects that we have studied were fixed, meaning that the observed levels of the treatment factor are the only levels of interest. In this model,

$$y_{ij} = \mu + \alpha_i + e_{ij},$$

the g values of α_i, $\alpha_1, \alpha_2, ..., \alpha_g$, are the only effects of interest, and e_{ij} is the only random term, e_{ij} has a normal distribution with mean 0 and variance σ_e^2 ($e_{ij} \sim N(0, \sigma_e^2)$). However, sometimes the treatment levels under study are only a sample of the possible treatment levels, and we wish to generalize our results to this larger set of levels. The cardboard carton machines example in section 11.1 is like this, where the ten machines in the study are only a sample of the population of 50 machines. For this type of study a more appropriate model is the random effects model:

$$y_{ij} = \mu + a_i + e_{ij},$$

Here the a_i terms are now random, $a_i \sim N(0\ ,\ \sigma_a^2\ )$, the e_{ij} terms have the same assumptions as before, and the a_i and e_{ij} are independent. Now the a_i 's are viewed as a random sample from a population of a_i ‘s, and the ANOVA H_0 is H_0: $\sigma_a^2 = 0$ vs. $H_a : \sigma_a^2 > 0$. Notice that our notation is a bit different from the text. The text uses the same symbol α_i for an effect whether it is fixed or random, while we are making a distinction between α_i and a_i.

How do you know if an effect is fixed or random?

1. How were the levels for the factor in question chosen?
2. Is it desired to generalize the results to levels that weren't used?

Implications for random effects

1. Generally you are not interested in doing multiple comparisons. There may be interest in estimating σ_a^2 , and σ_e^2 , or functions of them, like ratios.

2. The denominator of the F statistic may change (this occurs in more complex designs, like those shown in Displays 11.2 and 11.3).

Analysis of random effects models

The null hypothesis, H_0: $\sigma_a^2 = 0$ vs. $H_a : \sigma_a^2 > 0$ in the completely randomized (single-factor) design can be tested by the ANOVA F statistic just as with the fixed effects model, as can be seen from an examination of the expected mean squares. The text obtains estimators of the variance components by equating observed mean squares to their expected values, then solving for the variance components. This method, often called the method of moments, can encounter problems such as negative variance component estimates. Instead, we will use REML (restricted maximum likelihood, similar to maximum likelihood) estimators that are computed using Proc MIXED in SAS or various packages in R.

For multifactor random effects models, as seen in Displays 11.2 and 11.3, the table of expected mean squares must be examined to create the appropriate F statistic for each term. The key idea is that the expected value of the mean squares of the numerator and denominator of the F statistic must be equal under the null hypothesis. If an exact F test cannot be found, an approximate F test must be constructed.